6.2B Double Angle Identities

Special cases of the addition identities occur when the two angles are equal. In this instance, $A = B$, and therefore the expression $A+B$ could be replaced with $A+A$ or simply $2A$. This results in the following double angle identities:

$\sin(2A) = \sin(A+A)$ $= \sin A\cos A + \cos A\sin A$ $= 2\sin A\cos A$	$\cos(2A) = \cos(A+A)$ $= \cos A\cos A - \sin A\sin A$ $= \cos^2 A - \sin^2 A$	$\tan(2A) = \tan(A+A)$ $= \dfrac{\tan A + \tan A}{1-\tan A\tan A}$ $= \dfrac{2\tan A}{1-\tan^2 A}$

The identity for $\cos(2A)$ has two other forms:

$\sin^2 A = 1-\cos^2 A$ $\qquad$ $\cos^2 A = 1-\sin^2 A$

$\cos(2A) = \cos^2 A - \sin^2 A$ $= \cos^2 A - (1-\cos^2 A)$ $= 2\cos^2 A - 1$	$\cos(2A) = \cos^2 A - \sin^2 A$ $= (1-\sin^2 A) - \sin^2 A$ $= 1 - 2\sin^2 A$

Double Angle Identities
* $\sin 2x = 2\sin x\cos x$ $\cos 2x = \cos^2 x - \sin^2 x$ $= 2\cos^2 x - 1$ $= 1 - 2\sin^2 x$ * $\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}$

Example 1: Write each of the following as a single trigonometric ratio:

a) $30\sin A\cos A$

$15 \times \sin 2x = 15 \times 2\sin x\cos x$

$15\sin 2x = 30\sin x\cos x$

$\boxed{15\sin 2A} = 30\sin A\cos A$

b) $\cos^2 5x - \sin^2 5x$

$\cos 2A = \cos^2 A - \sin^2 A$

$\cos(2\cdot 5x) = \cos^2(5x) - \sin^2(5x)$

$\boxed{\cos(10x)}$

c) $10\sin 3x\cos 3x$

$5\sin 2x = 5\cdot 2\sin x\cos x$

$5\sin 2x = 10\sin x\cos x$

$5\sin(2\cdot 3x) = 10\sin(3x)\cos(3x)$

$\boxed{5\sin(6x)}$

d) $3 - 6\sin^2 4x$

$3\cos 2x = 3(1-2\sin^2 x)$

$3\cos 2x = 3-6\sin^2 x$

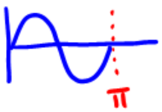
$3\cos(2\cdot 4x) = 3-6\sin^2(4x)$

$\boxed{3\cos 8x}$

Example 2: Consider the expression $\frac{1-\cos 2x}{\sin 2x}$.

a) What are the non-permissiblevalues? when $\sin 2x = 0$

① solve $\sin 2x = 0$



period $= \frac{2\pi}{b} = \frac{2\pi}{2} = \pi$

$x \neq 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi, \ldots$

$x \neq n \cdot \frac{\pi}{2}$

or ② solve an identity for $\sin 2x = 0$

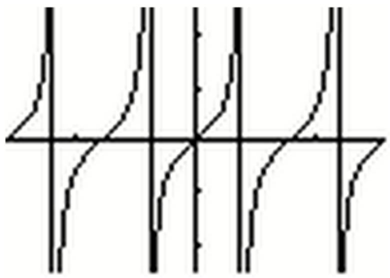
$2\sin x \cos x = 0$

$\sin x = 0$ $\quad$ $\cos x = 0$

$x \neq 0, \pi, 2\pi, \ldots$ $\quad$ $x \neq \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \ldots$

$x \neq n\frac{\pi}{2}$

b) Graph this expression on the interval $[-2\pi, 2\pi]$. What primary trigonometric function does this graph resemble?



looks like $y = \tan x$

c) Simplify the expression so that there is only one primary trigonometric ratio involved. Does this make seem reasonable given your result from part b?

$$\frac{1-\cos 2x}{\sin 2x} = \frac{1-(1-2\sin^2 x)}{2\sin x \cos x}$$

$$= \frac{\cancel{2}\sin^{\cancel{2}} x}{\cancel{2}\cancel{\sin x}\cos x} \quad \sin x$$

$$= \frac{\sin x}{\cos x}$$

- we chose an identity that would cancel out the 1 in $1 - 2\cos\theta$

which is equivalent to $\tan x$.

Example 3: Find the amplitude and the period of the graph of $y = 4\underbrace{\sin 2x \cos 2x}+3$.

$2 \cdot \sin 2x = 2 \cdot 2\sin x \cos x$

$2\sin 2x = 4\sin x \cos x$

$2\sin(2 \cdot 2x) = 4\sin(2x)\cos(2x)$

$+3 \quad\quad +3$

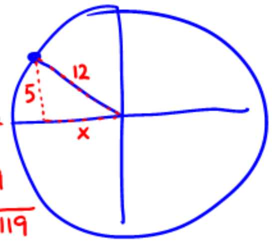
$y = 2\sin(4x) + 3$

amplitude $= 2$

period $= \frac{2\pi}{b} = \frac{2\pi}{4} = \frac{\pi}{2}$

Example 4: Given that $\sin\theta = \frac{5}{12}$ and that θ terminates in quadrant II, determine an exact value for

Use general circle.



$x^2+5^2=12^2$

$x^2=119$

$x=-\sqrt{119}$

$\sin\theta = \frac{y}{r} = \frac{5}{12}$

$\cos\theta = \frac{x}{r} = \frac{-\sqrt{119}}{12}$

a) $\sin 2\theta = 2\sin\theta\cos\theta$

$2\left(\frac{5}{12}\right)\left(-\frac{\sqrt{119}}{12}\right) = \frac{-10\sqrt{119}}{144} = \frac{-5\sqrt{119}}{72}$

b) $\cos 2\theta = 1-2\sin^2 x$

$= 1-2\left(\frac{5}{12}\right)^2$

$1-2\left(\frac{25}{144}\right) \rightarrow 1-\frac{50}{144} = \frac{144}{144}-\frac{50}{144}$

$= \boxed{\frac{94}{144}}$

c) $\sin\left(2\theta+\frac{\pi}{2}\right) = \sin A\cos B + \cos A\sin B$

$= \sin 2\theta\cos\frac{\pi}{2} + \cos 2\theta\sin\frac{\pi}{2}$

$= \left(\frac{-5\sqrt{119}}{72}\right)(0) + \left(\frac{94}{144}\right)(1)$

$= \frac{94}{144}$ in lowest terms $\frac{47}{72}$

Example 5: Simplify each of the following to a single trigonometric function:

a) $\frac{\sin 2x}{2\cos x} = \frac{\cancel{2}\sin x\cancel{\cos x}}{\cancel{2}\cancel{\cos x}}$ $= \sin x$ HW p 306 # 3-5, 11-13, 15, 16	b) $\frac{\sin^2 x}{\sin 2x} = \frac{\cancel{\sin^2 x}^{\sin x}}{2\cancel{\sin x}\cos x}$ $= \frac{1\ \sin x}{2\cos x}$ $= \frac{1}{2}\tan x$	c) $\frac{\cos 2x+1}{\sin 2x} = \frac{2\cos^2 x - 1 + 1}{2\sin x\cos x}$ $= \frac{\cancel{2}\cancel{\cos^2 x}^{\cos x}}{\cancel{2}\sin x\cancel{\cos x}}$ $= \frac{\cos x}{\sin x} = \cot x$