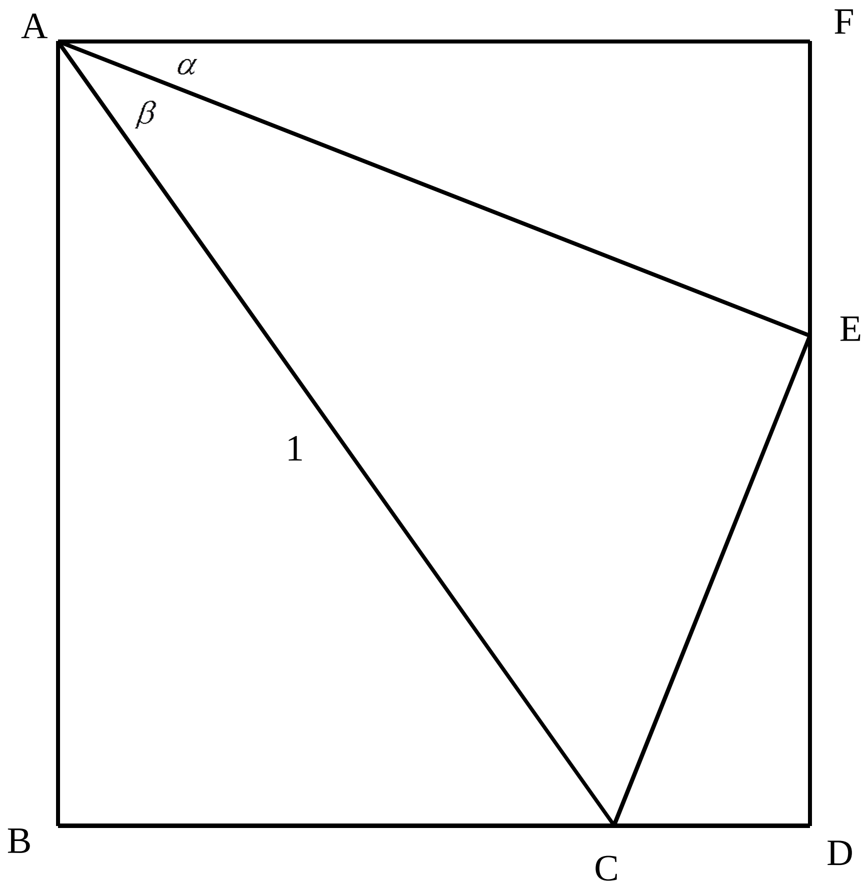
` *6.2A Warmup*

Given rectangle ABDF with ΔAEC drawn in such a way so that $\angle AEC$ is a right angle. Set the length of AC to equal one.

a) Express each of the other angles in terms of α and β.
b) Why is $EC = \sin\beta$ and $AE = \cos\beta$? Determine expressions for the lengths of each of the other sides in the diagram.



What identity does this diagram suggest for

i) $\sin(\alpha+\beta)$

ii) $\cos(\alpha+\beta)$

iii) $\tan(\alpha+\beta)=\dfrac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)}$

6.2A Sum and Difference Identities

The identities you have just discovered are called the angle sum identities. The angle difference identity for $\sin(A-B)$ can be obtained by rewriting this as $\sin(A+(-B))$ and then using $\cos(\theta)=\cos(-\theta)$ and $\sin(\theta)=-\sin(-\theta)$.

$\sin(A-B)=\sin(A+(-B))$ $=\sin A\cos(-B)+\cos A\sin(-B)$ $=\sin A\cos B-\cos A\sin B$	$\cos(A-B)=\cos(A+(-B))$ $=\cos A\cos(-B)-\sin A\sin(-B)$ $=\sin A\cos B+\sin A\sin B$

The preceding explorations lead to the following sum and difference identities.

Sum and Difference Identities	
$\sin(A+B)=\sin A\cos B+\cos A\sin B$ $\sin(A-B)=\sin A\cos B-\cos A\sin B$	$\tan(A+B)=\dfrac{\tan A+\tan B}{1-\tan A\tan B}$
$\cos(A+B)=\cos A\cos B-\sin A\sin B$ $\cos(A-B)=\cos A\cos B+\sin A\sin B$	$\tan(A-B)=\dfrac{\tan A-\tan B}{1+\tan A\tan B}$

Example 1: Express the following as a trigonometric function of a single angle:

a) $\sin\pi\cos\frac{\pi}{5}-\cos\pi\sin\frac{\pi}{5}$

$\sin(A-B)=\sin A\cos B-\cos A\sin B$

all A's are replaced by π

all B's are replaced with $\pi/5$

$\sin\left(\pi-\frac{\pi}{5}\right)\rightarrow\sin\left(\frac{4\pi}{5}\right)$

b) $\cos 32^0\cos 15^0+\sin 32^0\sin 15^0$

$\cos(A-B)=\cos A\cos B+\sin A\sin B$

$\cos(32^\circ-15^\circ)$

$\cos(17^\circ)$

Example 2: Simplify $\sin(x+\pi)+\cos\left(x-\frac{\pi}{2}\right)$ $=-\sin x+\sin x=0$

$\sin(A+B)=\sin A\cos B+\cos A\sin B$

$=\sin x\cdot\cos\pi+\cos x\sin\pi$

$=(\sin x)(-1)+(\cos x)(0)$

$=-\sin x$

$\cos(A-B)=\cos A\cos B+\sin A\sin B$

$=\cos x\cos\frac{\pi}{2}+\sin x\sin\frac{\pi}{2}$

$=(\cos x)(0)+(\sin x)(1)$

$=\sin x$

Example 3: Consider the identity $\sin(\frac{\pi}{2} - x) = \cos x$.

a) Verify the identity numerically, when $x = \frac{\pi}{6}$ without a calculator.

$$\sin\left(\frac{\pi}{2} - x\right) \stackrel{?}{=} \cos x$$

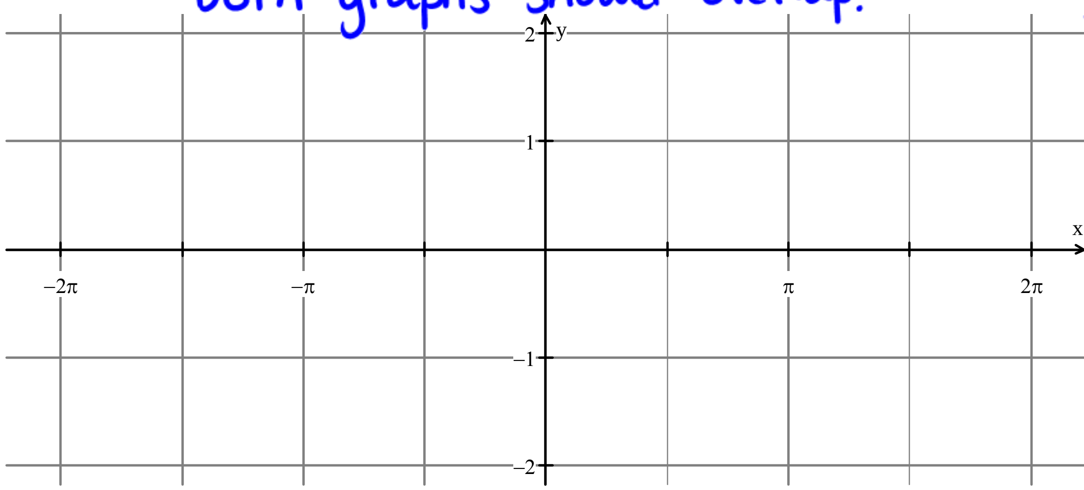
$$\sin\frac{\pi}{2}\cos x - \cos\frac{\pi}{2}\sin x$$

$$(1)(\cos x) - (0)(\sin x)$$

$$\cos x$$

b) Verify the identity graphically.

both graphs should overlap.



$y_1 = \sin(\frac{\pi}{2} - x)$

$y_2 = \cos x$

c) Use a difference identity to show why this is an aidentity.

Example 4: Use the fact that $15^0 = 60^0 - 45^0$ and a difference identity to find the **exact** value of $\cos 15^0$ and $\sin 15^0$.

$$\cos 15^\circ = \cos(60^\circ - 45^\circ)$$

$$= \cos 60 \cos 45 + \sin 60 \sin 45$$

$$= \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right)$$

$$= \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}} = \frac{1+\sqrt{3}}{2\sqrt{2}}$$

$$\sin(60 - 45)$$

$$= \sin 60 \cos 45 - \cos 60 \sin 45$$

$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right)$$

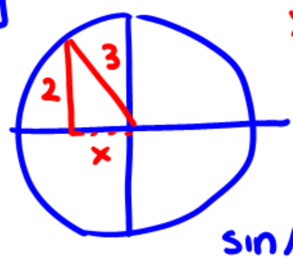
$$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

General Circle instead of Unit Circle.

$\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{y}{x}$

Example 5: If $\sin A = \frac{2}{3}$ and $\cos B = -\frac{3}{5}$ and both $\angle A$ and $\angle B$ are in Quadrant 2, evaluate

$\boxed{A}$



$x^2 + 2^2 = 3^2$

$x^2 = 5$

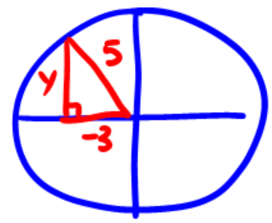
$x = -\sqrt{5}$

$\sin A = \frac{2}{3}$

$\cos A = -\frac{\sqrt{5}}{3}$

$\tan A = -\frac{2}{\sqrt{5}}$

$\boxed{B}$



$y^2 + (-3)^2 = 5^2$

$y^2 = 16$

$y = 4$

$\sin B = \frac{4}{5}$

$\cos B = -\frac{3}{5}$

$\tan B = -\frac{4}{3}$

i) $\cos(A - B)$

ii) $\sin(A + B)$

iii) $\tan(A - B)$

$\cos(A-B)$

$\cos A \cos B + \sin A \sin B$

$\left(-\frac{\sqrt{5}}{3}\right)\left(-\frac{3}{5}\right) + \left(\frac{2}{3}\right)\left(\frac{4}{5}\right)$

$\frac{3\sqrt{5}}{15} + \frac{8}{15}$

$\frac{3\sqrt{5}+8}{15}$

$\underline{\underline{\sin(A+B)}}$

$\underline{\underline{\tan(A-B)}}$

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Example 5: Use angle sum or difference identities to see if the following are true:

a) $\sin x = -\cos\left(x - \frac{3\pi}{2}\right)$

b) $\sin(x + \pi) = -\sin x$