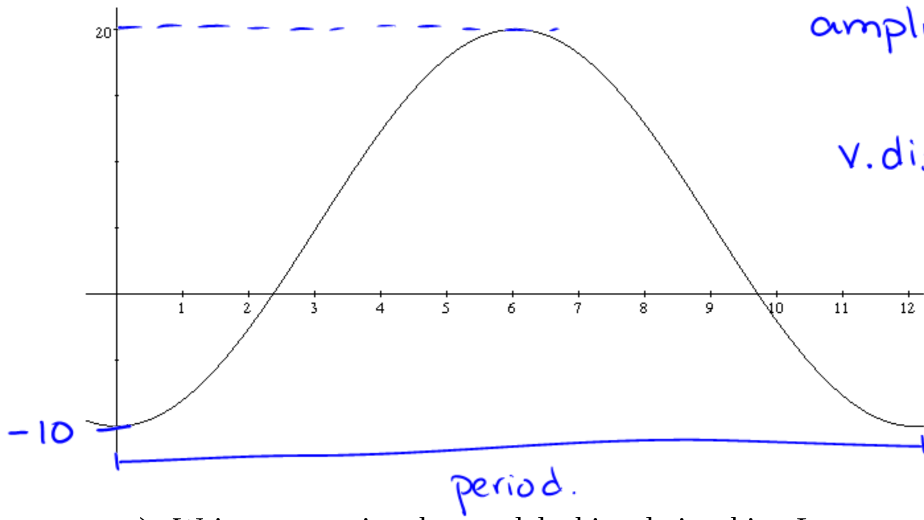
5.4B Modeling Real Situations Using Trigonometric Functions

Example 1. The average monthly temperature for a location in Northern BC as a function of month number can be modelled using the equation $y = a\cos b(x - c) + d$. The highest average monthly temperature is 20°C and the lowest average monthly temperature is –10°C, which occurs in January (month 0) with an annually repeating pattern. The sketch of the graph that models this relationship is shown below.

Total diff = $20 - {}^{-}10 = 30$

amplitude $\boxed{15}$.



v.displ : $\dfrac{20 + {}^{-}10}{2} = \boxed{5}$

period = $\dfrac{2\pi}{b}$

$12 = \dfrac{2\pi}{b} \therefore \boxed{b = \dfrac{2\pi}{12}}$

a) Write an equation that models this relationship. Let your variable represent the day number (as opposed to the month number)

$T = -15\cos\left(\frac{2\pi}{12} \cdot x\right) + 5$

average temperature where
x = month number.

$T = -15\cos\left(\frac{2\pi}{365}x\right) + 5$

average temperature where
x represents day number.

b) What might you expect the average temperature to be on the 60^{th} day of the year?

*radian mode

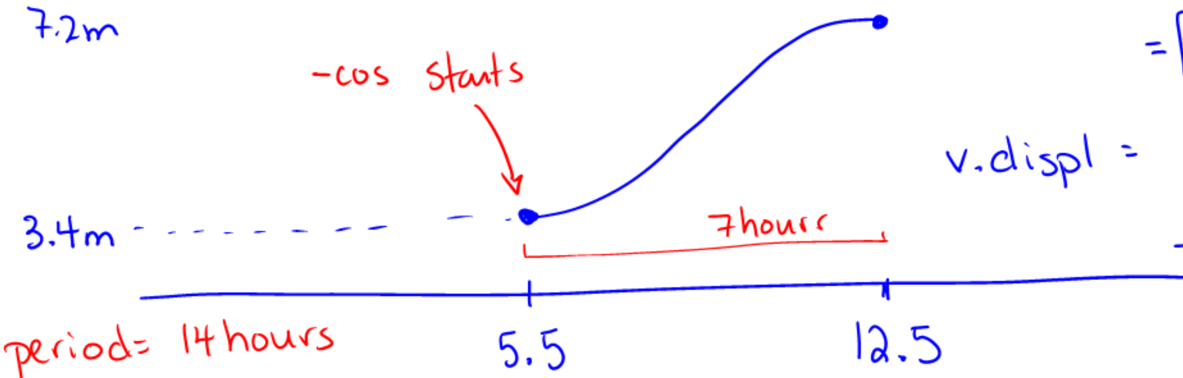
$x = 60$ $\quad T = -15\cos\left(\frac{2\pi}{365}(60)\right) + 5$

$T = -2.7°C$

Example 2: Tides can be thought of as a roughly periodic rise and fall of the ocean's water. On a certain day at the Tsawwassen ferry terminal, a low tide occurred of 3.4 m occurred at 5:30 AM and a high tide of 7.2 m occurred at 12:30 PM. If we assume that the relation between the depth of water and the time is a sinusoidal function,

amplitude $= \frac{7.2-3.4}{2} = \boxed{1.9}$

a) What is a sinusoidal function that describes the tide flow?



v.displ $= \frac{3.4+7.2}{2} = \boxed{5.3}$

full period = 14 hours

$P = \frac{2\pi}{b}$

$14 = \frac{2\pi}{b} \quad \therefore \boxed{b = \frac{2\pi}{14}}$

hours after midnight

$D = -1.9\cos\left(\frac{2\pi}{14}(x-5.5)\right)+5.3$

note: $D = -1.9\cos\left(\frac{2\pi(x-5.5)}{14}\right)+5.3$

$x =$ # hours after midnight.

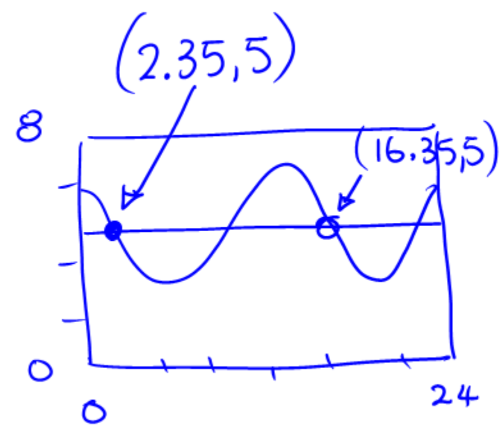
b) What will the height of the tide be at 2 PM.

$D = -1.9\cos\left(\frac{2\pi(14-5.5)}{14}\right)+5.3$

$\boxed{D = 6.8\text{ m}}$

c) Estimate one of the times when the water is 5 m deep.

$\underbrace{5}_{Y_1} = \underbrace{-1.9\cos\left(\frac{2\pi(x-5.5)}{14}\right)+5.3}_{Y_2}$

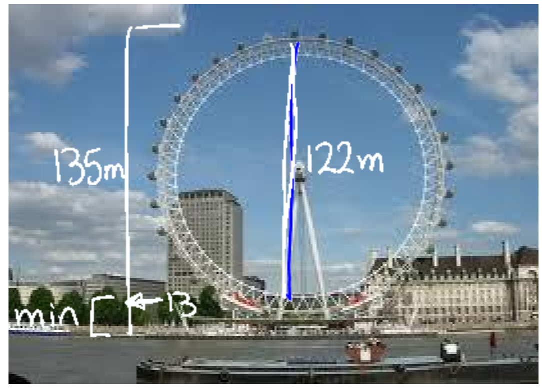


at 2.35 hours after midnight, the water is 5 m deep. This is at 2:21 am.

$\frac{35}{100} \times 60 = 21$ minutes

This depth also occurs at 4:21 pm.

Example 2. The London Eye has a diameter of 122 m and a height of 135 m. It completes one rotation in 30 minutes. Because it rotates so slowly, passengers are able to board and exit at the bottom by just walking on or off.



max=135

min=13

a) Write a sinusoidal function to describe a person's height at any time?

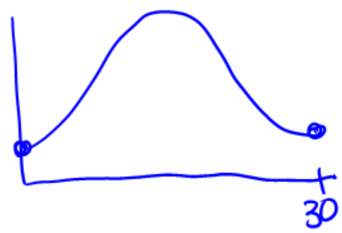
amplitude: $\frac{135-13}{2} = 61\text{m}.$

v.displ : $\frac{135+13}{2} = 74\text{m}$

period = 30 minutes.

$P = \frac{2\pi}{b} \therefore b = \frac{2\pi}{30}$

$30 = \frac{2\pi}{b}$



$-\cos$

$h = -61\cos\left(\frac{2\pi}{30}(x)\right) + 74.$

b) How long after boarding would it take you to reach a height of 100 m?

$$\underbrace{100}_{Y_1} = \underbrace{-61\cos\left(\frac{2\pi}{30}(x)\right) + 74}_{Y_2}.$$

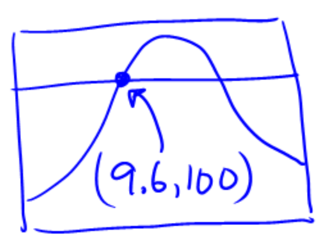
window: $0 < x < 30$ (represented by period)

$0 < y < 135$ (represents height)

9.6 minutes

$.6 \times 60 = 36$

9 min 36 seconds.



Example 4: Write a sinusoidal function that models the average monthly temperature in Vancouver given the data below. (Given are the average monthly high temperatures)

Month	Temp (°C)	Month	Temp (°C)	Month	Temp (°C)
1	6	5	16	9	18
2	8	6	19	10	13
3	9	7	22	11	9
4	12	8	22	12	6

This data can be entered and plotted on your calculator. You can then enter your equation, and see how well it approximates the data.

① Enter Data

STAT > 1: edit

months in L1

temps in L2

② Window Settings

$0 < x < 12$ (months)

$0 < y < 25$ (temp

③ Plot points.

STAT PLOT

2nd Y=

- turn on stat plot

④ Find equation.

STAT CALC C: sin reg.

SinReg L1 , L2

a: 7.76

b= 0.57

c= -2.52

d= 13.94

$y = 7.76\sin(.57(x - 2.52)) + 13.94$