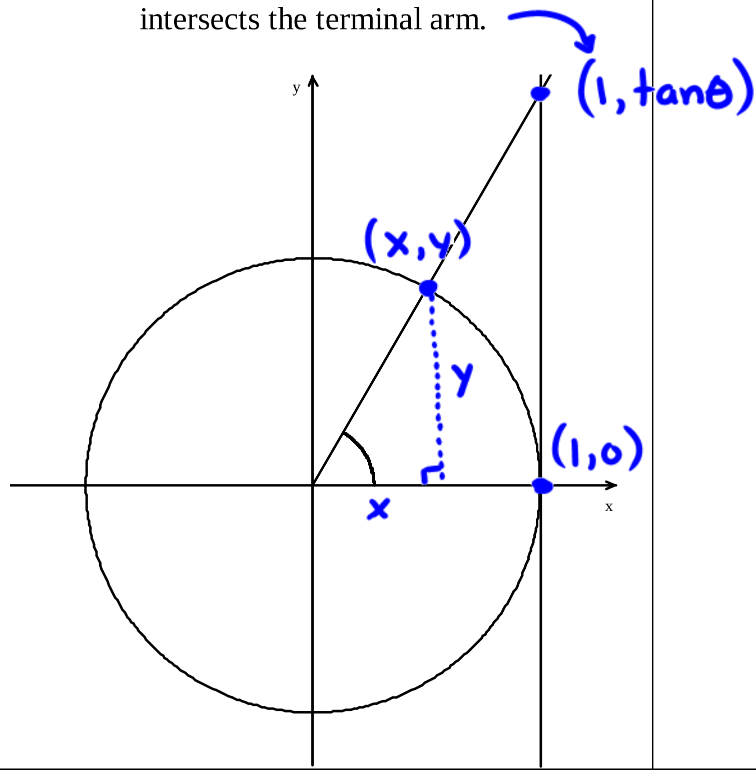
5.3 Graphing the Tangent Function

Recall the different definitions of $\tan\theta$ that you now have

Right triangle	Circular definition	Using sine and cosine
$\tan\theta = \frac{opp}{adj}$	$\tan\theta = \frac{y}{x}$	$\tan\theta = \frac{\sin\theta}{\cos\theta}$

Unit Circle – where the tangent to the circle intersects the terminal arm.



Slope Definition

$\tan\theta =$ slope of the terminal arm

$\frac{\text{rise}}{\text{run}} = \frac{y}{x}$

$m = \frac{y}{x} = \tan\theta$

To graph $y = \tan\theta$, we will begin by making a table of values using what we know about the special triangles. Give exact values and decimal approximations to 2 decimal places.

$\theta_R = 30^\circ$

θ	0	30^0	45^0	60^0	90^0	120^0	135^0	150^0	180^0
$\tan\theta$	0	.58	1	1.73	n.p	-.58	-1	-1.73	0

Note that after you reach $\theta = 180^0$, the values for $\tan\theta$ start to repeat.

θ	180^0	210^0	225^0	240^0	270^0	300^0	315^0	330^0	360^0
$\tan\theta$	0	+.58	+1	+1.73	np	-.58	-1	-1.73	0

$\tan\theta$ repeats every 180° $\therefore$ period for $\tan\theta$ is π^R or 180° unlike $\sin\theta$ or $\cos\theta$ which have a period of 360°

The most interesting behaviour of the tangent function occurs as θ approaches angles such as $90^\circ, 270^\circ, 450^\circ$, etc. where $\tan\theta$ is undefined.

Let's investigate the values of $\tan\theta$ for angles very close to 90°. For convenience, we will use degree measurement.

Fill in the values for $\tan\theta$ in the tables below.

θ	$\tan\theta$
70°	2.747
80°	5.67
89°	57.3
89.9°	573
89.99°	5729
89.999°	57296
89.9999°	572958

θ	$\tan\theta$
90.0001°	−572958
90.001°	−57296
90.01°	−5729
90.1°	−573
91°	−57.3
100°	−5.67
110°	−2.747

Questions:

1) What happens to $\tan\theta$ as θ gets closer and closer to 90°?

becomes a large positive number

2) What happens to $\tan\theta$ as θ "jumps over" the 90° threshold to 90.0001°?

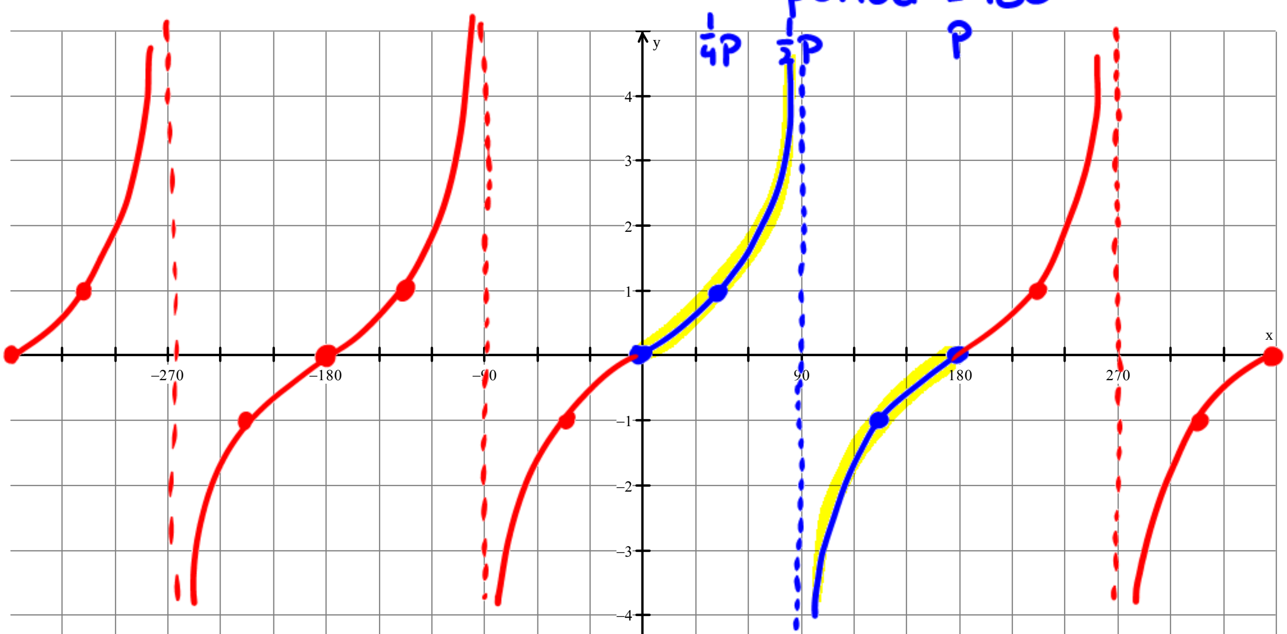
it switches to be a large negative number.

This is asymptotic behaviour around $\theta = 90^\circ$ where the tangent is undefined.

there is an asymptote when $\theta = 90^\circ$

Using this information, sketch the graph of $y = \tan\theta$

Period = 180°



tanθ

starts	0
$\frac{1}{4}P$	1
$\frac{1}{2}P$	asymp
$\frac{3}{4}P$	−1
P	0

domain : $x \in \mathbb{R},\ x \neq 90^\circ + n180^\circ$ n is an integer

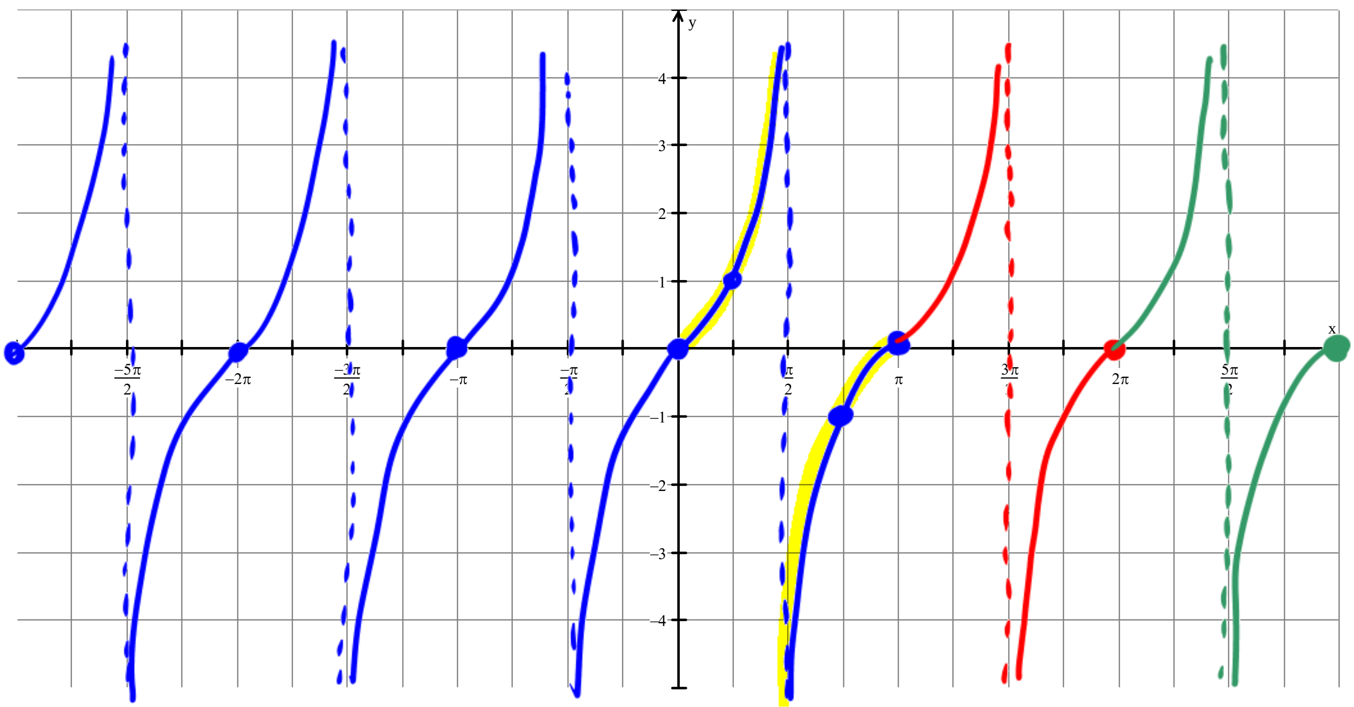
What is the period of the tangent function? 180° or π radians.

What is the amplitude of the tangent function? no amplitude

What are the equations of the asymptotes? $x = 90° + 180°n$ where n is an integer.

What are the x-intercepts? $x = 180°n$

Construct the graph of $y = \tan x$, where x is any real number.



Features of the Graph of $y = \tan x$

<u>Wavelength:</u> (period) = 180° or π radians.

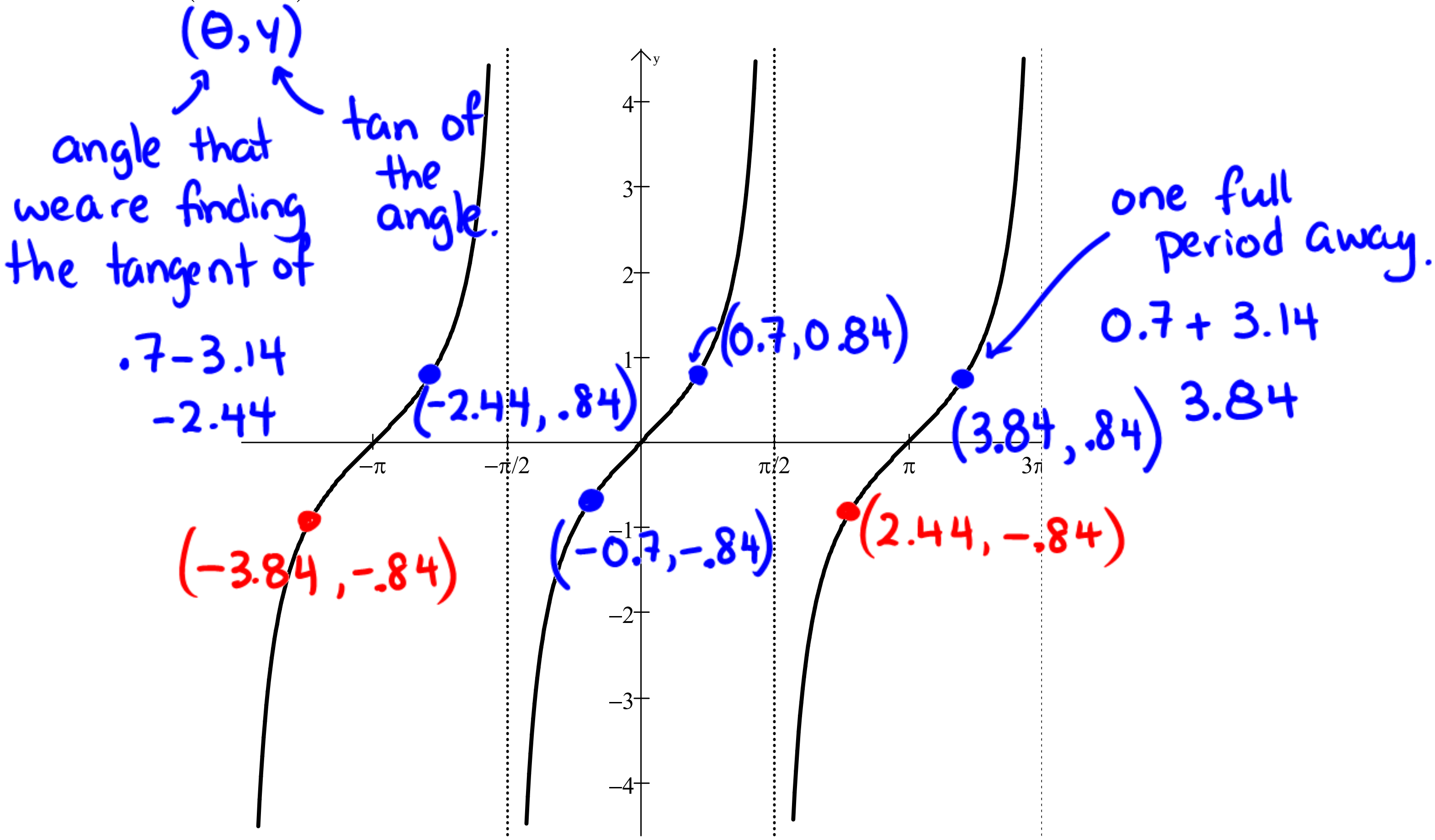
<u>Range:</u> $y \in \mathbb{R}$.

<u>Domain:</u> $\boxed{x \in \mathbb{R};\ x \neq \frac{\pi}{2} + n\pi}$

<u>Asymptotes:</u> $x = 90° + n180°$ or $x = \frac{\pi}{2} + n\pi$

The Graph of $y = \tan\theta$

The point $(0.7, 0.84)$ is on the graph of $y = \tan\theta$. Find the coordinates of 4 other points on the graph:



How many solutions to the equation $\tan\theta = -0.84$ does the graph above show?

3 solutions shown, but there are really an infinite number.

What are these solutions?

$\theta = -3.84, -0.7, 2.44$

How would you describe all the solutions to $\tan\theta = 0.84$?

$\theta = -0.7 + n\pi$ where n is any integer.

Does the equation $\tan\theta = k$ where k is any real number always have a solution?

range: $y \in \mathbb{R}$, you can have any value k and there will be a solution for $\tan\theta = k$.