$y = 3\cos\frac{2}{5}x$

maximum $3$

minimum $-3$

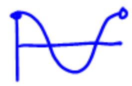
amplitude $= 3$

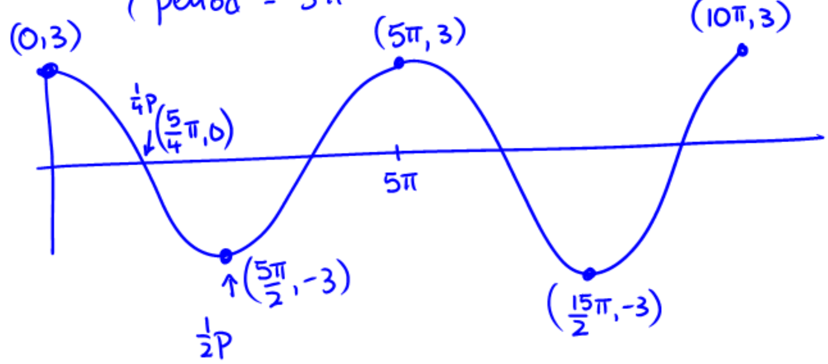
$$\text{period} = \frac{2\pi}{b}$$

$$= \frac{2\pi}{\left(\frac{2}{5}\right)}$$

$$\text{period} = 5\pi$$

$y = \cos x$





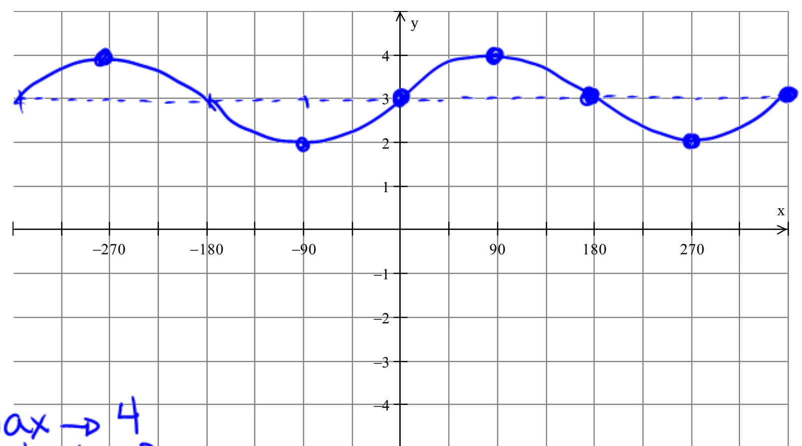
# *5.2Transforming Sinusoidal Functions*

For each of the following, identify the transformation that occurs:

| | Original function | New function | Transformation |
|---|---|---|---|
| a | $y = x^2$ | $y = x^2 + 3$ | moves up 3 |
| b | $y = f(x)$ | $y = f(x) - 2$ | moves down 2 |
| c | $y = \sqrt{x}$ | $y = \sqrt{x+2}$ | moves left 2 |
| d | $y = f \;\; y = f(x)$ | $y = f(x-6)$ | moves right 6 |

The graphical translations that transform functions can also be applied to the graphs of sine and cosine.

Predict what the graphs of $y = \sin\theta + 3$ will look like, and then graph using technology:




max → 4
central → 3
min → 2

Open the graphic calculator website at http://www.desmos.com/calculator
Entering in the function $y = \sin\theta + d$ will let you create a "slider" that will vary the value of *d*.

What happens to the graph as *d* varies?

Graph moves up/down
central axis/middle = d

Determine the maximum and minimum values for the function you graphed above. How does this compare to the value of *d*?

***Vertical Displacement*** $y = \sin\theta + d$

When graphing a sinusoidal function, a vertical translation is also called the vertical displacement and is represented by the parameter *d*.

- If *d* > 0, then the graph is translated up |d| units. $\Big\}$ central axis at "d"
- If *d* < 0, then the graph is translated down |d| units.

The vertical displacement can also be calculated as:

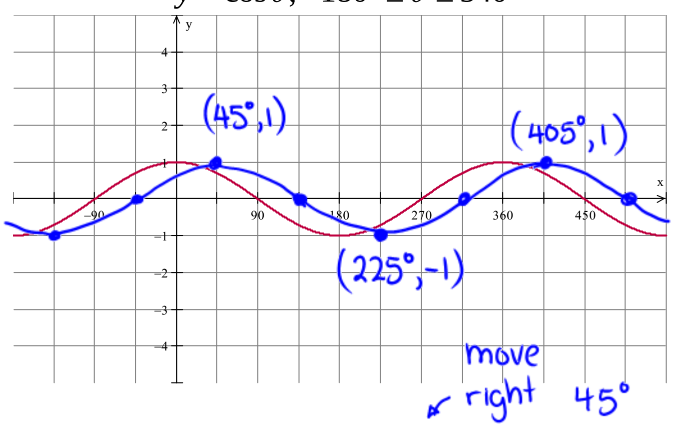
$d = \dfrac{max + min}{2}$ "taking average or finding the middle of max and min"

Using the Desmos Graphic Calculator, graph $y = \cos(x - c)$ and add a slider for the parameter *c*.
What effect does changing the value of *c* have on the graph of the function?

moves graph left / right without changing the period.

Graph each of the following functions on the grids provided:

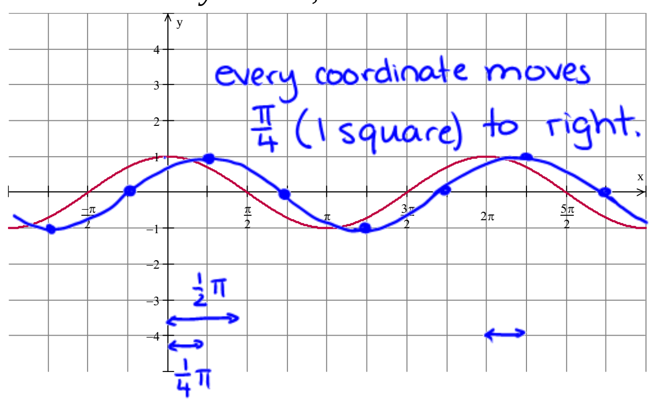
$y = \cos\theta; -180^\circ \le \theta \le 540^\circ$




Sketch the graph of $y = \cos(x - 45^\circ)$

note: the period did not change.

$y = \cos x; -\pi \le x \le 3\pi$




Sketch the graph of $y = \cos\left(\theta - \frac{\pi}{4}\right)$

Note the transformation is the same whether you are measuring in degrees or radians.

***Phase Shift*** $y = \sin(\theta - c)$

When graphing a sinusoidal function, a horizontal translation is also called the

___phase shift___ and is represented by the parameter *c*.

- If *c* > 0, then the graph is translated ___to the right.___.
- If *c* < 0, then the graph is translated ___to the left.___.

Note the effect that the minus sign has on the equation of the function.

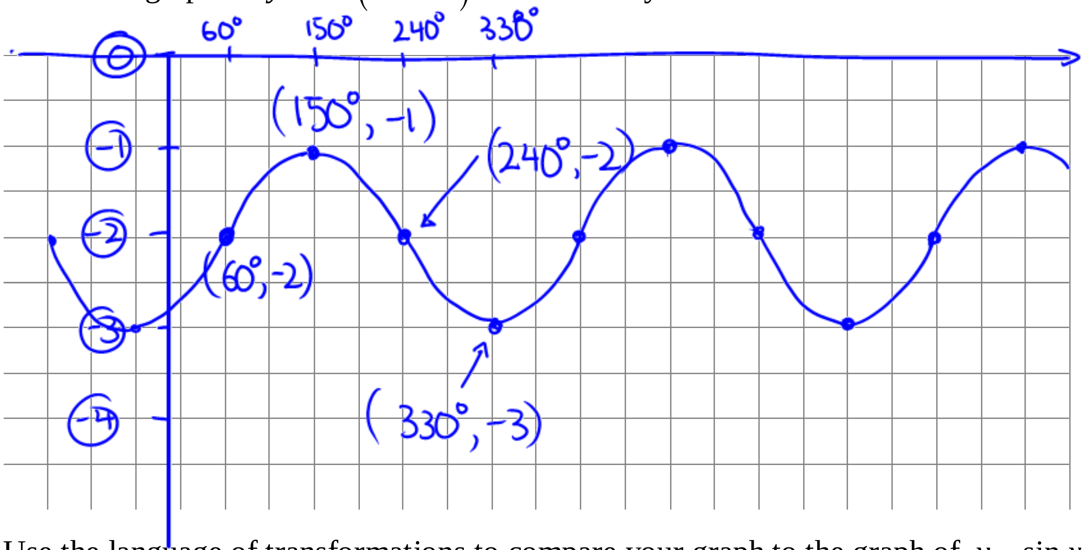
For example, if *c* = 4, then the equation of the transformed function is:

$y = \sin(\theta - 4)$ if $c = -2$ $y = \sin(\theta - -2)$

or $y = \sin(\theta + 2)$

| | | | |
|---|---|---|---|
| 0 | 0° | → | 60° |
| $\frac{1}{4}$P | 90° | → | 150° |
| $\frac{1}{2}$P | 180° | → | 240° |
| $\frac{3}{4}$P | 270° | → | 330° |
| P | 360° | → | 420° |

Example 1:
Sketch the graph of $y=\sin\left(x-60^\circ\right)-2$ for two cycles:



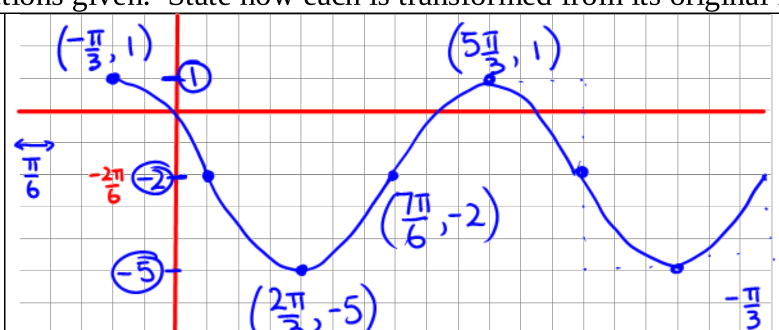
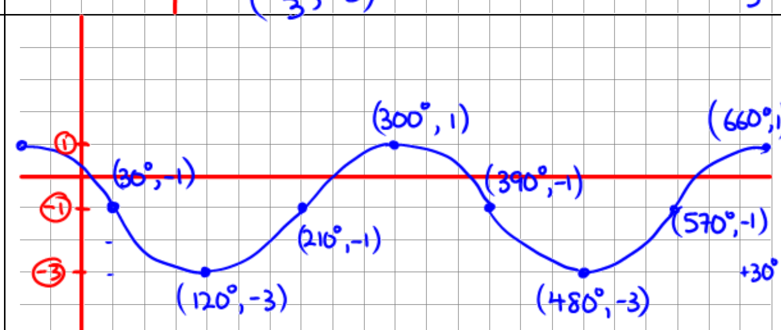
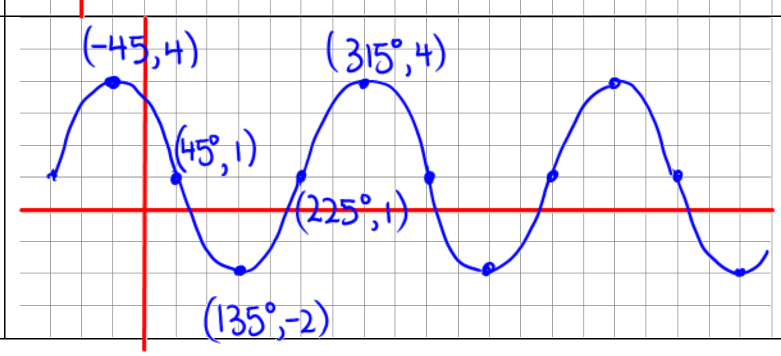

Domain:
$x \in \mathbb{R}$

Range:
min max
$-3 \le y \le -1$

Use the language of transformations to compare your graph to the graph of $y=\sin x$.

phase shift of +60° and a vertical displacement of -2.

Example 2:
Sketch each of the functions given. State how each is transformed from its original function:

| | | |
|---|---|---|
| $y=3\cos\left(\theta+\frac{\pi}{3}\right)-2$<br>v.disp = -2<br>phase shift = $-\frac{\pi}{3}$ |  | max: 1<br>mid: -2<br>min: -5 |
| $y=-2\sin\left(x-30^\circ\right)-1$<br>v.disp = -1<br>phase shift: 30°<br>amplitude = 2 |  | max = 1<br>mid = -1<br>min = -3 |
| $y=3\cos\left(x+45^\circ\right)+1$<br>V.disp = +1<br>amplitude = 3<br>phase shift = -45° |  | max: 4<br>mid: 1<br>min: -2 |



| | | | | |
|---|---|---|---|---|
| 0 | $\frac{1}{4}$P | $\frac{1}{2}$P | $\frac{3}{4}$P | P |
| 0 | $\frac{\pi}{2}$ | $\pi$ | $\frac{3\pi}{2}$ | $2\pi$ |
| $-\frac{\pi}{3}$ | $\frac{\pi}{6}$ | $\frac{2\pi}{3}$ | $\frac{7\pi}{6}$ | $\frac{5\pi}{3}$ |



| | | | | |
|---|---|---|---|---|
| 0 | $\frac{1}{4}$P | $\frac{1}{2}$P | $\frac{3}{4}$P | P |
| 0 | 90° | 180° | 270° | 360° |
| 30° | 120° | 210° | 300° | 390° |



| | | | | |
|---|---|---|---|---|
| 0 | $\frac{1}{4}$P | $\frac{1}{2}$P | $\frac{3}{4}$P | P |
| 0 | 90° | 180° | 270° | 360° |
| -45° | 45° | 135° | 225° | 315° |

$y = a\cos b(x-c)+d$

Example 3:

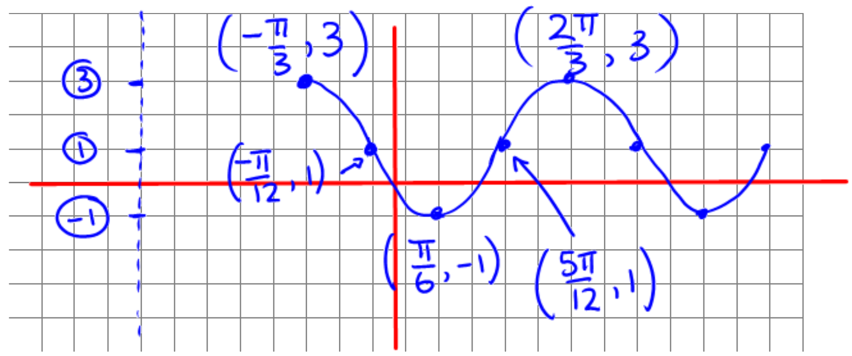
Consider the function: $y = 2\cos\left(2x+\frac{2\pi}{3}\right)+1$. Determine:

b is inside bracket

$y = 2\cos 2\left(x+\frac{\pi}{3}\right)+1$

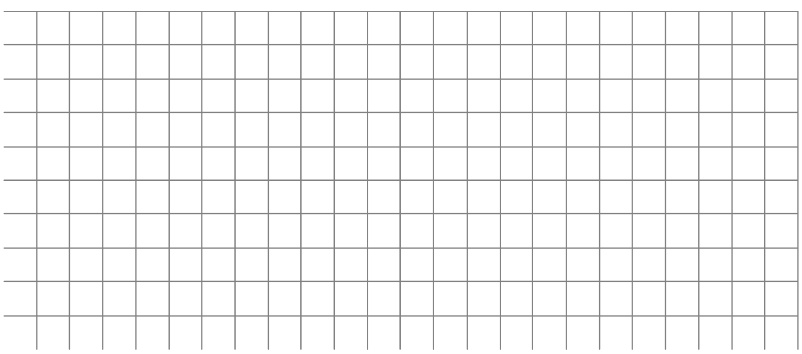
| | | | | | |
|---|---|---|---|---|---|
| Domain: | $x \in \mathbb{R}$ | Amplitude | 2 | V. Disp. | +1 |
| Range: | $-1 \le y \le 3$ | Period | $\frac{2\pi}{(2)} = \pi$ | Phase Shift | $-\frac{\pi}{3}$ |

Sketch the function:




Note: This function can also be graphed using 5 key points:

| Start (max) | middle | minimum | middle | 1 full period |
|---|---|---|---|---|
| $0 - \pi/3$<br>$-\pi/3$ | $\pi/4 - \pi/3$<br>$-\pi/12$ | $\pi/2 - \pi/3$<br>$\pi/6$ | $3\pi/4 - \pi/3$<br>$5\pi/12$ | $\pi - \pi/3$<br>$2\pi/3$ |

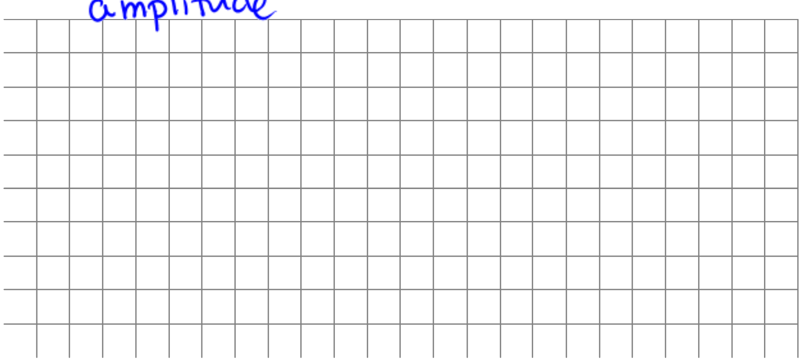


Example 4:

v.displ　　phase shift

The graph of $y = \cos x$ is translated 3 units up and $\frac{\pi}{3}$ units to the right. It has been stretched vertically by a factor of 2 and reflected in the $x$-axis. Determine the equation of this function and sketch the graph:

amplitude



$y = -2\cos\left(x-\frac{\pi}{3}\right)+3$