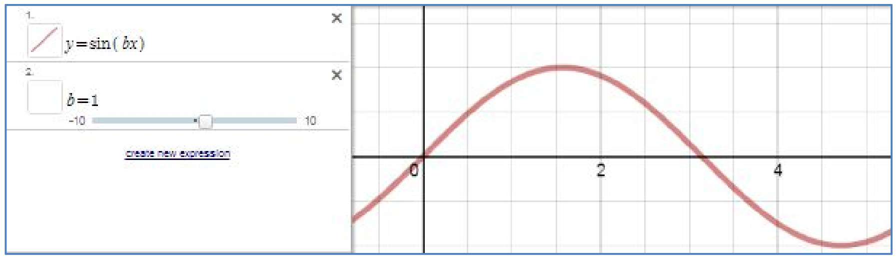
5.1b Horizontal Stretches of sine and cosine

Recall the transformation that occurs when the function $y = f(x)$ is mapped to $y = f(bx)$:

a horizontal stretch by a factor of $\frac{1}{b}$

Open the graphic calculator website:
http://www.desmos.com/calculator
Entering in the function $y = \sin b\theta$ will let you create a "slider" that will vary the value of *b*.



Note the following as *b* varies.

Amplitude: does not change.

y-intercept: does not change

vertical displacement: middle does not change

x-intercept: changes.

period: changes

What effect does *b* have on the function, $y = \sin bx$?

- only changes period which also changes the x-intercepts.

Since transformations apply to the domain and range of a function, the stretch caused by *b* can also be applied to the period:

This is the interval of one cycle for $y = \sin x$ or $y = \cos x$ measured in radians

replace x with bx

$0 \le x \le 2\pi$ shows one full cycle for $y = \sin x$ } replace x with bx

$0 \le |bx| \le 2\pi$ to make $y = \sin(bx)$

$\frac{0}{b} \le \frac{bx}{b} \le \frac{2\pi}{b}$

1 full period.

$0 \le x \le \frac{2\pi}{b}$

This is the interval of one cycle for $y = \sin x$ or $y = \cos x$ measured in degrees

$0 \le x \le 360^\circ$ ← period for $y = \sin x$ or $y = \cos x$

$0 \le |bx| \le 360^\circ$

$\frac{0}{b} \le \frac{bx}{b} \le \frac{360^\circ}{b}$

$0 \le x \le \frac{360^\circ}{b}$

note: $\frac{360^\circ}{b}$ is equivalent to $360 \times \frac{1}{b}$

* amplitude is always a positive number

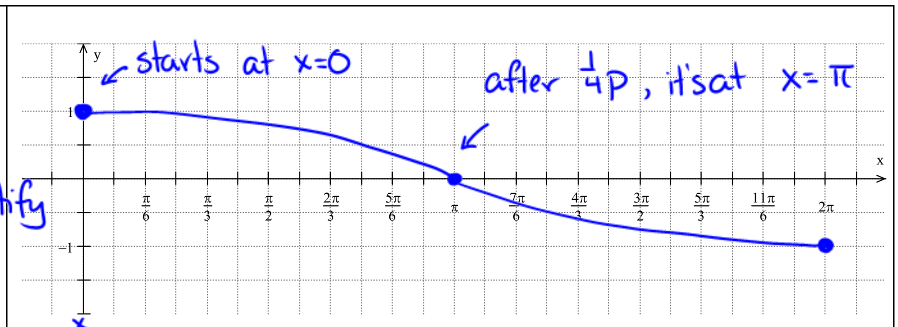
Example 1 What is the period and amplitude for each function?

	Function	Period	Amplitude
a)	$y = -5\cos 3x$	$\frac{360^\circ}{3}$ or $\frac{2\pi}{3}$	5
b)	$y = 2\sin\frac{1}{2}x$ $\quad \frac{360^\circ}{\frac{1}{2}}$	720° or 4π	2
c)	$y = -2\cos 3x$	$\frac{360^\circ}{3}$ or $\frac{2\pi}{3}$	2

Example 2 Without using the graphing calculator, sketch the graph of $y = \cos\frac{1}{2}x$ for $-2\pi \le x \le 2\pi$.

$y = \cos\frac{1}{2}x$

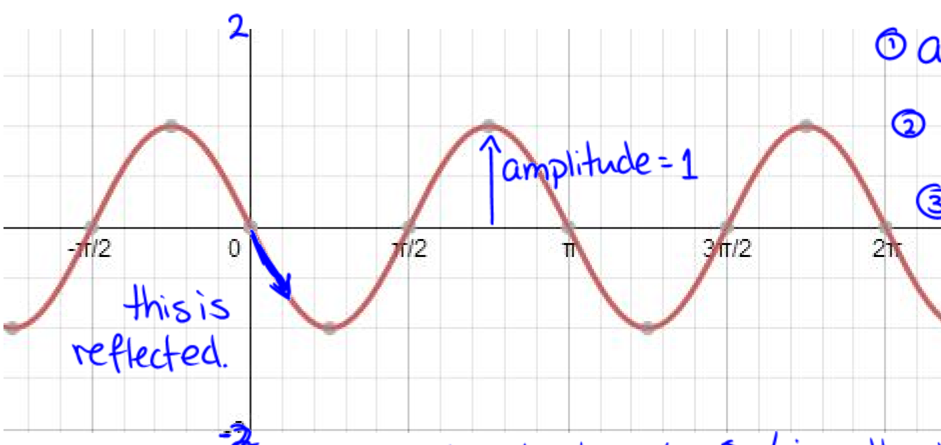
important to identify period + 4 other points.



starts	$\frac{1}{4}$ period	$\frac{1}{2}$ period	$\frac{3}{4}$ p.	full period
0	π	2π	3π	4π

Example 3

Write an equation for the graph shown below:



① amplitude

② period

③ sin or cos.

sine because it starts at 0 / in the middle.

period $= \pi$

$\frac{2\pi}{b} =$ period

$\frac{2\pi}{b} = \pi$

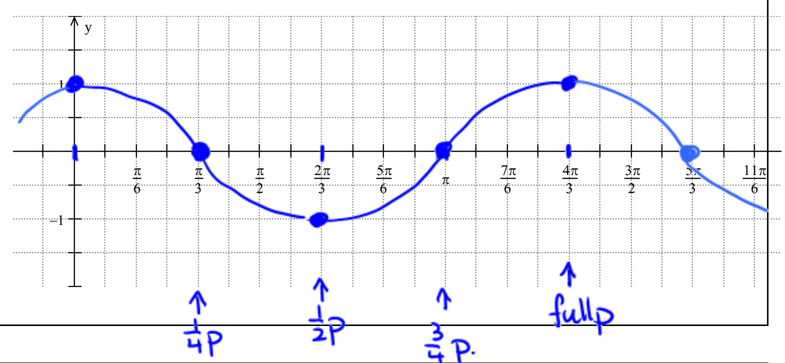
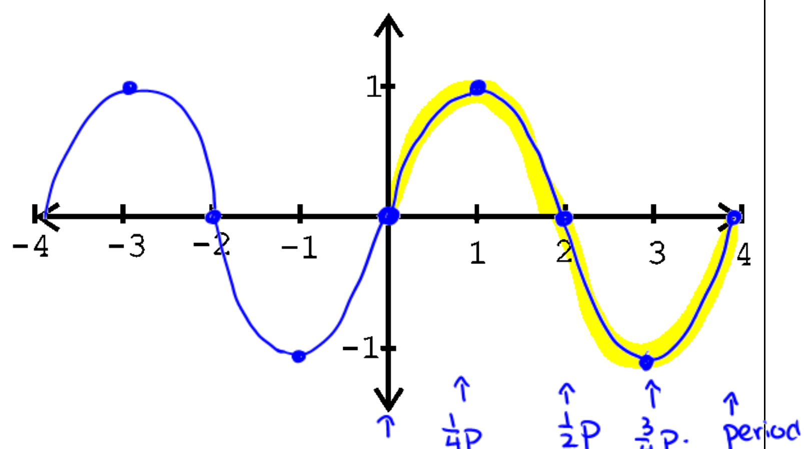
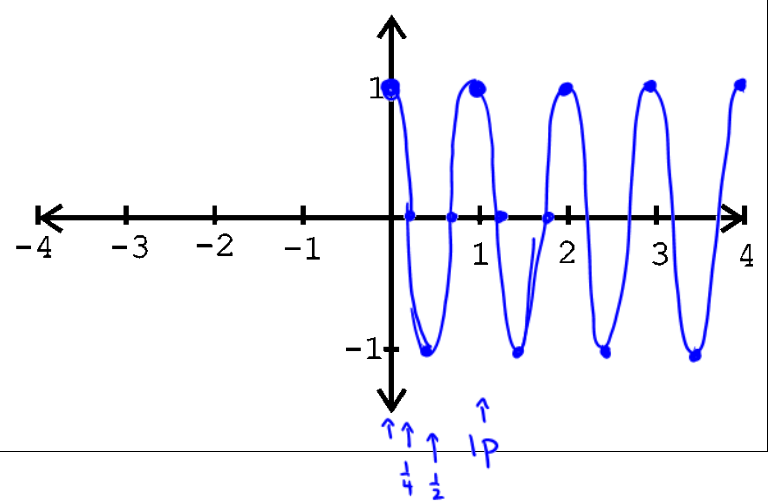
$b = \frac{2\pi}{\pi} = 2$

$y = -\sin 2x$

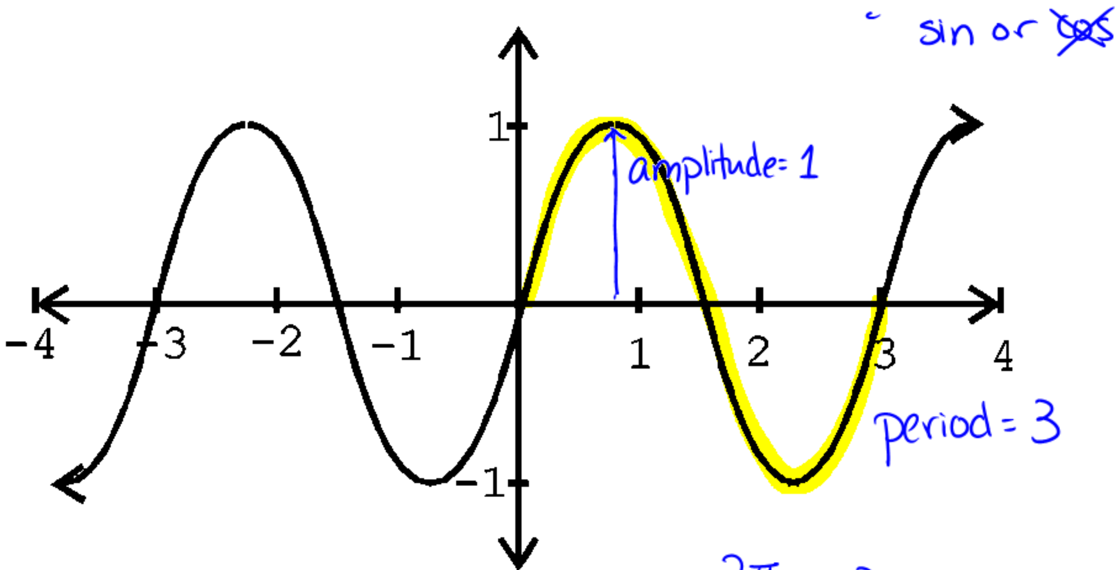
our graph is reflected.

Example 4

Predict the period of the following trigonometric function. Graph the functions.

Trig Function	Period	Graph
a) $y=\cos\frac{3}{2}x$ $\frac{2\pi}{(\frac{3}{2})} = 2\pi \times \frac{2}{3}$	$P=\frac{4\pi}{3}$ $\frac{1}{2}P=\frac{2\pi}{3}$ $\frac{1}{4}P=\frac{\pi}{3}$	
b) $y=\sin\frac{2\pi}{4}x$ $P=\frac{2\pi}{(\frac{2\pi}{4})}$ $=2\pi\times\frac{4}{2\pi}=4$	$P=4$ $\frac{1}{2}P=2$ $\frac{1}{4}P=1$	
c) $y=\cos 2\pi x$ $P=\frac{2\pi}{(2\pi)}=1$	$1P=1$ $\frac{1}{2}P=\frac{1}{2}$ $\frac{1}{4}P=\frac{1}{4}$	

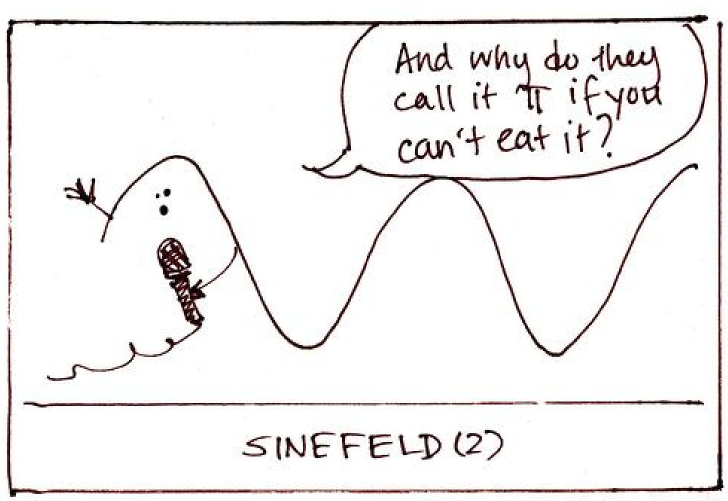
Example 5
Write a trigonometric equation to match the graph below:



$$\frac{2\pi}{b} = 3$$

$$b = \frac{2\pi}{3}$$

$$y = \sin\left(\frac{2\pi}{3}x\right)$$



SINEFELD (2)

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