1.1 Translating Graphs of Functions

1. Comparing the graphs of $y = f(x)$ and $y = f(x) + k$ [or $y - k = f(x)$]

(a) Complete the following tables of values. Graph and label each of the functions on the grid

$y = |x|$

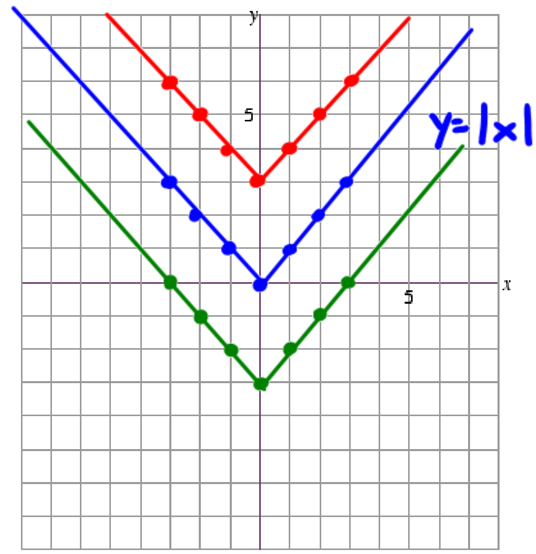
x	y
–3	3
–2	2
–1	1
0	0
1	1
2	2
3	3

$y = |x| + 3$

x	y
–3	6
–2	5
–1	4
0	3
1	4
2	5
3	6

$y = |x| - 3$

x	y
–3	0
–2	-1
–1	-2
0	-3
1	-2
2	-1
3	0



b) How are each of the following graphs obtained from the graph of $y = |x|$?

i) $y = |x| + 3$ moved up 3 units

ii) $y = |x| - 3$ moved down 3

c) In general, how is the graph of $y = |x| + k$ obtained from the graph of $y = |x|$

i) when $k > 0$? eg $k=5$ $y=|x|+5$ up k units

ii) when $k < 0$? eg $k=-2$ $y=|x|+-2$ or $y=|x|-2$ down k units

d) The graph of $y = f(x) + k$ [or $y - k = f(x)$] is obtained when the graph of $y = f(x)$ undergoes a vertical shift (or translation) of k units. translation = move

If $k > 0$, the graph of $y = f(x)$ is translated up to obtain the graph of $y = f(x) + k$ [or $y - k = f(x)$]. If $k < 0$, the graph of $y = f(x)$ is translated down to obtain the graph of $y = f(x) + k$ [or $y - k = f(x)$].

eg $y=f(x)+3$ moves up $y=f(x)-3$ moves down

Note: The notation $y - k = f(x)$ is often used instead of $y = f(x) + k$ to emphasize that the parameter k involves a translation in the y-direction only. For example, instead of $y = |x| + 3$, we could write $y - 3 = |x|$.

$y-3=f(x)$ is same as $y=f(x)+3$ both show a translation up

2. **Comparing the graphs of $y = f(x)$ and $y = f(x-h)$**

(a) Complete the following tables of values. Use the table of values to graph and label each of the functions on the grid.

$y = x^2$

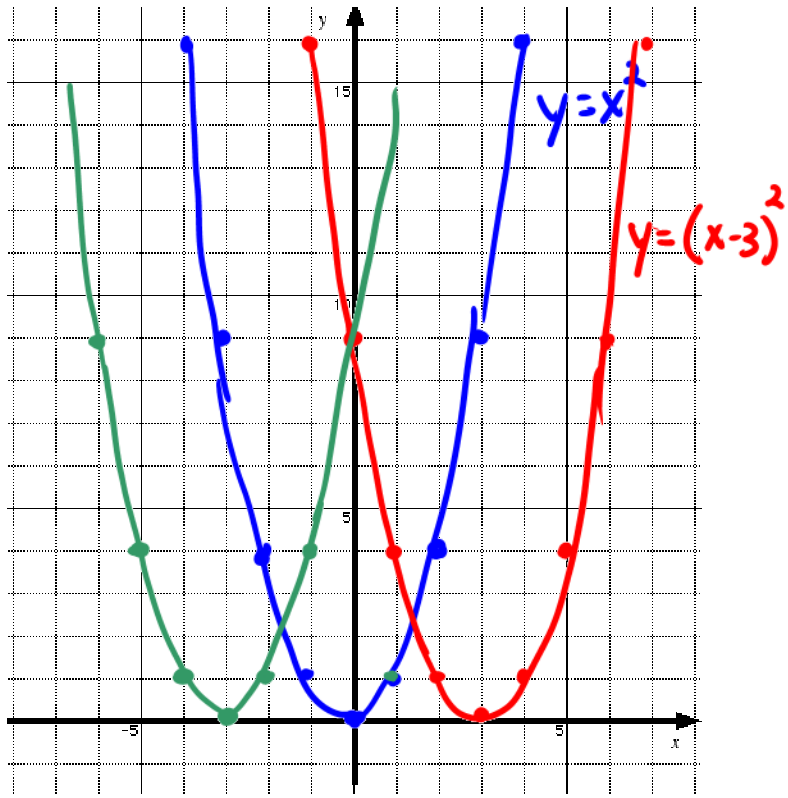
x	y
–4	16
–3	9
–2	4
–1	1
0	0
1	1
2	4
3	9
4	16

$y = (x-3)^2$

x	y
–1	16
0	9
1	4
2	1
3	0
4	1
5	4
6	9
7	16

$y = (x+3)^2$

x	y
–7	16
–6	9
–5	4
–4	1
–3	0
–2	1
–1	4
0	9
1	16



b) How are each of the following graphs obtained from the graph of $y = x^2$?

i) $y = (x-3)^2$

translation 3 units right.

ii) $y = (x+3)^2$

translated 3 units left

c) In general, how is the graph of $y = (x-h)^2$ obtained from the graph of $y = x^2$

i) when $h > 0$?

h=4

translation right.

$y=(x-4)^2$

ii) when $h < 0$?

h=-4

results in $y=(x--4)^2$

or

$y=(x+4)^2$

d) The graph of $y = f(x-h)$ is obtained when the graph of $y = f(x)$ undergoes a horizontal shift (or translation) of h units. If $h > 0$, the graph of $y = f(x)$ is translated to the right to obtain the graph of of $y = f(x-h)$.
If $h < 0$, the graph of $y = f(x)$ is translated to the left to obtain the graph of $y = f(x-h)$.

Note that the equation $y = (x+3)^2$ can be written in the form $y = (x-h)^2$ as $y = (x - {}^{-}3)^2$. So in this case, $h = {}^{-}3$ and the translation of $y = x^2$ is 3 units to the left.

3. Horizontal and vertical translations

By translating the graph of $y = |x|$, sketch the graph of $y - 3 = |x + 2|$. → $y = \underbrace{|x+2|}_{\text{left 2}} \underbrace{+3}_{\text{up3}}$

To obtain the graph of $y - 3 = |x + 2|$, all points on the graph of $y = |x|$ will be translated

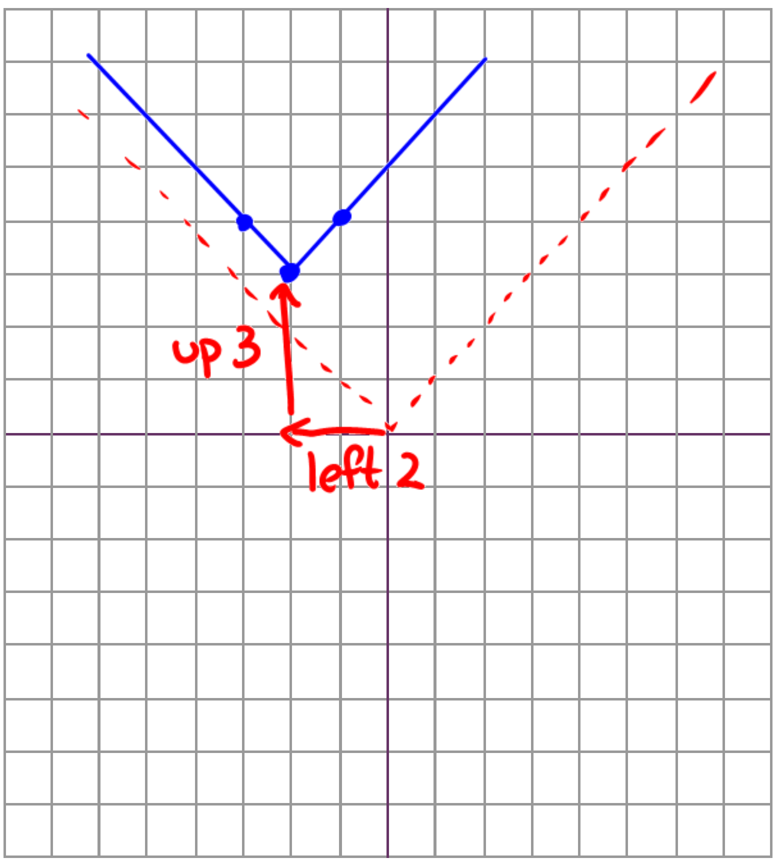
horizontally: left 2

vertically: up 3 .

Thus the point (0, 0) of $y = |x|$ will become the point (-2,3) of $y - 3 = |x + 2|$.

Likewise, the point (1, 1) of $y = |x|$ will become the point (-1,4) of $y - 3 = |x + 2|$, and the point (–1, 1) of $y = |x|$ will become the point (-3, 4) of $y - 3 = |x + 2|$.

*horizontal changes only affect "x"



Example 1:

Given the function $y = f(x)$, write the equation of the transformed function after each of the following translations.

a) a vertical translation 4 units down. $y = f(x) - 4$

b) a horizontal translation 5 units to the right. $y = f(x-5)$

c) a horizontal translation 3 units to the left and a vertical translation 6 units up. $y = f(x+3) + 6$

Example 2:

Describe how the graphs of the following functions can be obtained from the graph of $y = f(x)$.

a) $y = f(x+4)$ left 4

b) $y = f(x) - 5$ down 5

c) $y = f(x-2) + 3$ right 2 up 3

Example 3:

In each case below, the given point is transformed into a second point by a certain translation. Find the coordinates of the second point.

a) a horizontal translation 3 units to the left

$(4, -6) \rightarrow (1, -6)$

b) a vertical translation 5 units down

$(-3, -5) \rightarrow (-3, -10)$

c) a horizontal translation 4 units to the right and a vertical translation 6 units up

$(-7, 2) \rightarrow (-3, 8)$

Example 4:

In each case below, describe the translation that transforms the first point onto the second point.

a) $(5, -2) \rightarrow (5, 4)$ up 6 $y = f(x) + 6$

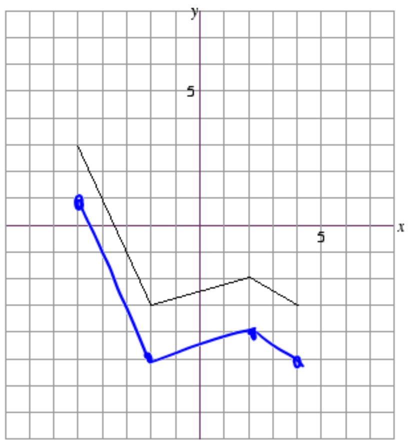
b) $(-6, -3) \rightarrow (5, -3)$ right 11 $y = f(x-11)$

c) $(4, -7) \rightarrow (-2, -5)$ 6 left 2 up $y = f(x+6) + 2$

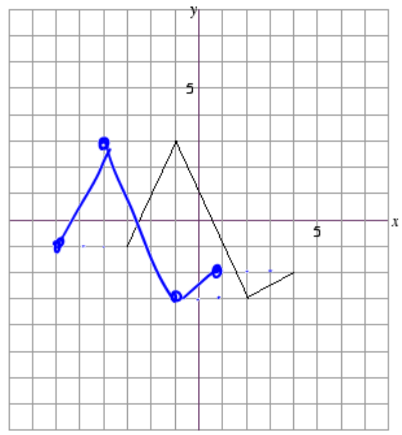
Example 5:

In each case below, a graph of $y = f(x)$ is shown. Sketch the graph of the translated function whose equation is given.

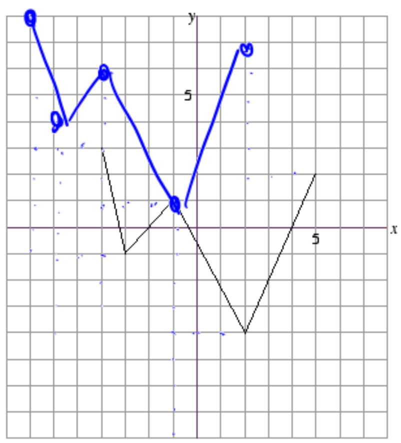
a) $y = f(x) - 2$



b) $y = f(x+3)$

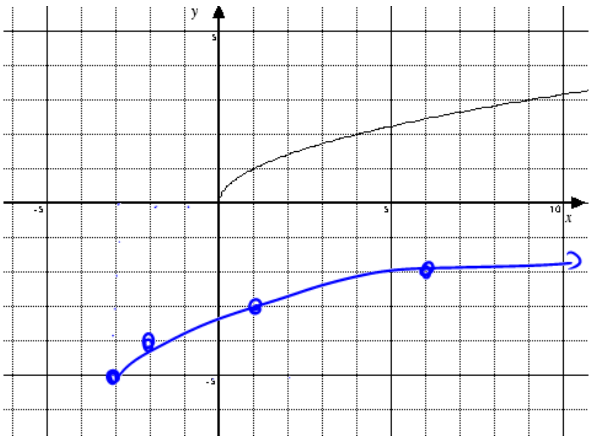


c) $y - 5 = f(x+3)$



Example 6:

Use the graph of $y = \sqrt{x}$ below to sketch the graph of $y + 5 = \sqrt{x+3}$.



Example 7:

The function $y = f(x)$ has x-intercepts of -6 and 10, y-intercept of -9, domain $\{x \geq -8\}$ and range $\{y \leq 2\}$

Give the same information for the functions defined below

		x-intercepts	y-intercept	domain	range
a) $y = f(x - 2)$	right 2	$-4, 12$	unknown	$x \geq -6$	$y \leq 2$
b) $y = f(x) + 3$	up 3	unknown	-6	$x \geq -8$	$y \leq 5$
c) $y = f(x + 1) - 4$	$x \to x+1$ $y \to y+4$	~~unknown~~	unknown	$x \geq -9$	$y \leq -2$

none because