

4.4B Trigonometric Equations

Factoring to solve trigonometric equations

Solve the following trigonometric equations in the specified domain. Where possible, give exact values. Otherwise give approximate values.

a) $\tan^2 \theta + 3 \tan \theta + 2 = 0$ $0 \leq \theta < 2\pi$

$$x = \tan \theta$$

$$x^2 + 3x + 2 = 0$$

$$(x+1)(x+2) = 0$$

$$x = -1 \text{ or } x = -2$$

$$\frac{S}{T} \mid \frac{A}{C}$$

$$\tan \theta = -1$$

special Δ



$$\theta_R = \frac{\pi}{4}$$

$$Q2: \theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4} \checkmark$$

$$Q4: \theta = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4} \checkmark$$

$$\tan \theta = -2$$



$$\theta_R = \tan^{-1}(2) = 1.11$$

$$Q2: \theta = \pi - 1.11 = 2.03$$

$$Q4: 2\pi - 1.11 = 5.17$$

$$\theta = \frac{3\pi}{4}, \frac{7\pi}{4}, 2.03, 5.17$$

b) $2 \sin^2 \theta - 3 \sin \theta = -1$ $0 \leq \theta < 360^\circ$

$$x = \sin \theta$$

$$2x^2 - 3x = -1$$

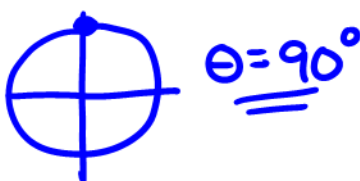
$$2x^2 - 3x + 1 = 0$$

$$2x^2 - x - 2x + 1 = 0$$

$$x(2x-1) - 1(2x-1) = 0$$

$$(x-1)(2x-1) = 0$$

$$\sin \theta = 1$$

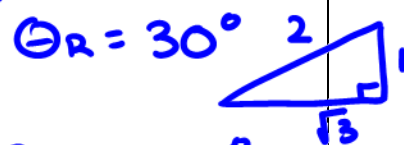


$$\theta = 90^\circ$$

$$\sin \theta = \frac{1}{2}$$



special triangle



$$Q1: \theta = 30^\circ$$

$$Q2: \theta = 150^\circ$$

$$\theta = 30^\circ, 90^\circ, 150^\circ$$

c) $2\cos^2 x + 5\cos x + 2 = 0$

Solve over the reals.

(General Solution)

$\cos x = y$

$2y^2 + 5y + 2 = 0$

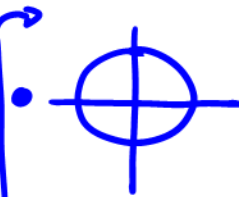
$2y^2 + y + 4y + 2 = 0$

$y(2y+1) + 2(2y+1) = 0$

$(y+2)(2y+1) = 0$

$(\cos x + 2)(2\cos x + 1) = 0$

$\cos x = -2$



this part has no solution!

$\cos x = -\frac{1}{2}$



special Δ



$x_R = \frac{\pi}{3}$

Q2: $x = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

Q3: $x = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$

$x = \frac{2\pi}{3} + n2\pi$

$x = \frac{4\pi}{3} + n2\pi$

d) $2\sin^2 x - \sin x - 2 = 0$

Solve over the reals.

$\sin x = a$

$2a^2 - a - 2 = 0$

cannot be factored

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-2)}}{2(2)}$

$x = 1.28$ or -0.781

$\sin x = 1.28$

this part has no solution!

$\sin x = -0.781$



not special Δ

$x_R = \sin^{-1}(.781)$

$x_R = 0.9$

Q3: $x = \pi + .9$

Q4: $x = 2\pi - .9$

$x = 4.04 + n2\pi$

$x = 5.38 + n2\pi$