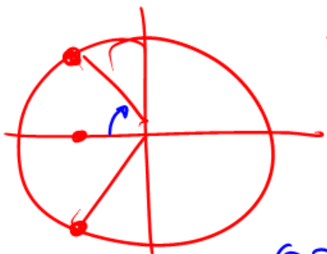
4.4A Warmup

1. You are asked to find θ such that $\cos\theta = -0.542$ with $0 \le \theta < 2\pi$. In terms of the unit circle what does this mean you are looking for? What are the values for θ?

- 2 angles because there are 2 places where $x = -.542$

- Q2 and Q3



Find reference angle

$\theta_R = \cos^{-1}(.542)$ ← $|-.542|$

$\theta_R = .998$

Q2: $\theta = \pi - 1$

$= 2.14$

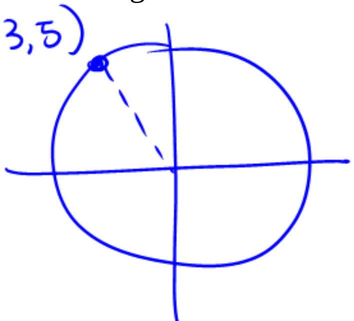
Q3: $\theta = \pi + 1$

$= 4.14$

$\theta = 2.14$ or 4.14

2. The point $Q(-3,5)$ lies on the terminal arm of an angle θ. Determine the exact value of each of the trigonometric ratios and the smallest positive value of θ in radians.

(-3,5)

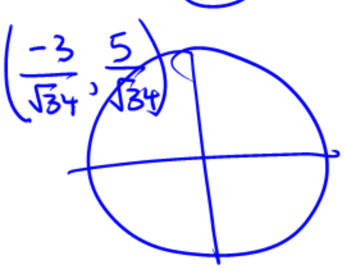


① Find radius using Pythag.

$(-3)^2 + 5^2 = r^2$

$r = \sqrt{34}$

②a redraw unit circle



$\sin\theta = \frac{5}{\sqrt{34}}$ $\quad \csc\theta = \frac{\sqrt{34}}{5}$

$\cos\theta = \frac{-3}{\sqrt{34}}$ $\quad \sec\theta = -\frac{\sqrt{34}}{3}$

$\tan\theta = -\frac{5}{3}$ $\quad \cot\theta = -\frac{3}{5}$

- given both x and y there is only 1 Q2 answer.

$\theta_R = \sin^{-1}\left(\frac{5}{\sqrt{34}}\right)$

$\theta_R = 1.03$

Q2: $\theta = \pi - 1.03$

$\underline{\underline{\theta = 2.11}}$

4.4A Trigonometric Equations

The following are examples of different types of equations you have solved, along with some examples of trigonometric equations

Algebraic Equations	Trigonometric Equations
$2x+5=16x-7$	$2\cos\theta+5=16\cos\theta-7$
$x^2=\frac{1}{2}$	$\sin^2\theta=\frac{1}{2}$
$x^2-5x-6=0$	$\cos^2\theta-5\cos\theta-6=0$
$\frac{3}{x}=\frac{7}{4}$	$\frac{3}{\cos\theta}=\frac{17}{4}$
$\sqrt{x-3}=5$	$\sqrt{\sec\theta-3}=5$

get sin/cos/tan by itself and then find angle

Solving trigonometric equations involves using both algebraic equation solving skills, along with knowledge of trigonometric functions. Additionally, trigonometric equations usually come with a restriction on the domain which then places a limit on the number of solutions.

Example 1: Solve the following trigonometric equations in the specified domain.

a) $2\sin\theta-5=-4\sin\theta-1$ $\quad 0\le\theta<2\pi$ (Round answers to 2 decimal places)

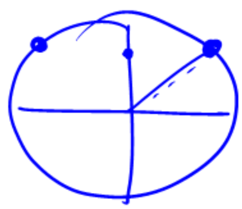
$+4\sin\theta \qquad +4\sin\theta$

$6\sin\theta-5=-1$

$+5 \qquad +5$

$6\sin\theta=4$

$\sin\theta=\frac{2}{3}$



Q1 and Q2

$\theta_R=\sin^{-1}\left(\frac{2}{3}\right)$

$=0.73$

Q1 $\theta=0.73$

Q2 $\theta=\pi-.73$

$=2.41$

$\theta=0.73,\ 2.41$ when $0\le\theta<2\pi$

(0.4, 0.1) 

b) $\sec\theta - 5 = 4\sec\theta - 12$ $\boxed{0 \le \theta < 360^0}$ (Round answers to 2 decimal places)

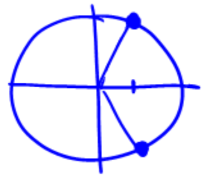
$-\sec\theta \quad -\sec\theta$

$+12 \quad +12$

$7 = 3\sec\theta$

$\sec\theta = \frac{7}{3}$

$\boxed{\cos\theta = \frac{3}{7}}$



$\theta_R = \cos^{-1}\left(\frac{3}{7}\right)$

$\theta_R = 64.6^\circ$

Q1 : $\theta = \underline{64.6^\circ}$

Q4 $\theta = 360 - 64.6^\circ$

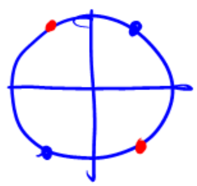
$= \underline{295.4^\circ}$

c) $3\tan^2\theta - 1 = 0$ $0 \le \theta < 2\pi$ (Give exact values)

$3\tan^2\theta = 1$

$\tan^2\theta = \frac{1}{3}$

$\tan\theta = \pm\frac{1}{\sqrt{3}}$



special triangle

30° or $\frac{\pi}{6}$

Q1: $\theta = \frac{\pi}{6}$ Q3: $\theta = \frac{7\pi}{6}$

Q2: $\theta = \frac{5\pi}{6}$ Q4: $\theta = \frac{11\pi}{6}$

Example 2. The General Solution to a Trigonometric Equation
Solve the following over the reals.

4.4a P211 #1, 3, 5, 6, 10, 11, 15, 16

a) $5\sin x + 1 = 0$

b) $\tan^2 x = 1$

c) $2\cos x \sin x + \cos x = 0$ (Give exact values)