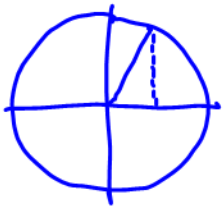
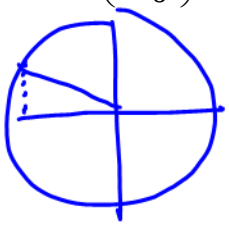
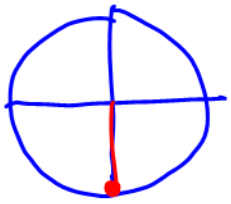
4.3B Warmup

1. Determine the approximate value of — use calculator

a) $\tan(8.2)$ -2.7737	b) $\csc(-91^0) = -1.0001$ $\sin(-91^\circ) = -.9998$
c) $\cos(-3.14)$ -1.000	d) $\sec(-89^0)$ $= 57.3000$ $\cos(-89^\circ) = 0.01745$

2. Determine the exact value of — use special triangles.

a) $\sin\left(\frac{85\pi}{3}\right) = \sin\left(\frac{13\pi}{3}\right) = \sin\left(\frac{\pi}{3}\right)$ $2\pi = \frac{6\pi}{3}$  $\sin\frac{85\pi}{3} = \frac{\sqrt{3}}{2}$	b) $\csc\left(-\frac{7\pi}{6}\right)$ $\sin\left(-\frac{7\pi}{6}\right) = \frac{1}{2}$  $\csc\left(-\frac{7\pi}{6}\right) = 2$
c) $\sin\left(\frac{47\pi}{2}\right) = \sin\left(\frac{3\pi}{2}\right)$ $2\pi = \frac{4\pi}{2}$  $\sin\left(\frac{47\pi}{2}\right) = -1$	d) $\tan\left(\frac{47\pi}{2}\right) = \frac{\sin\theta}{\cos\theta} = \frac{-1}{0}$ = undefined

3. If $P(-0.6, 0.8)$ lies on the terminal arm of θ on the unit circle, what is

a) $\cos\theta$ — $\cos\theta = -.6$

b) $\sin\theta$ — $\sin\theta = .8$

c) $\tan\theta$ — $\tan\theta = -\frac{.8}{.6} = -\frac{8}{6}$

Why do you know that $P(-0.6, 0.8)$ lies on the unit circle?

$x^2 + y^2 = r^2$

$(-.6)^2 + (.8)^2 = 1^2$ ✓ $\quad .36 + .64 = 1$ ✓

$\tan\theta = -\frac{4}{3}$

4. Is $\cos\left(\frac{5\pi}{6}\right) = \cos\left(150^0\right)$? Explain.

yes. $\frac{5\pi}{6}$ and 150° are coterminal angles in standard position / same angle.

① Find reference angle. - use inverse trig functions on a positive value

② Find the angle in standard position.

4.3B Finding the Angle

Example 1: Determine the measure of all angles that satisfy the given conditions.

a) $\cos\theta = -0.738$ $\quad 0 \le \theta < 2\pi$

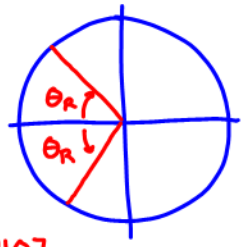
Give answers to the nearest hundredth of a radian.

x-coord negative in Q2 and Q3

$\cos\theta_R = +.738$

$\theta_R = \cos^{-1}(.738)$

$\theta_R = .7407$



Q2: $\theta = \pi - .7407$

$\theta = 2.4009$

Q3: $\theta = \pi + .7407$

$\theta = 3.8823$

Which quadrants will the answers be in?

How many answers will there be?

$0 \le \theta < 2\pi$ covers 1 full rotation.

There are 2 solutions in Q2 and Q3

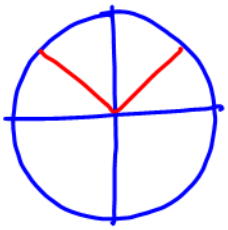
b) $\sin\theta = 0.315$ $\quad 0 \le \theta < 360^0$

Give answers to the nearest tenth of a degree.

$\sin\theta_R = 0.315$

$\theta_R = \sin^{-1}(.315)$

$\theta_R = 18.4^\circ$



Q1: $\theta = 18.4^\circ$

Q2: $\theta = 180 - 18.4^\circ$

$\theta = 161.6^\circ$

Which quadrants will the answers be in?

Q1 and Q2

How many answers will there be?

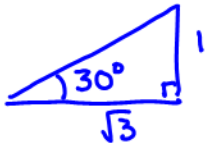
2 solutions in 1 rotation.

S | A
T | C

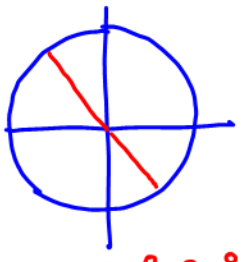
c) $\tan\theta = -\frac{1}{\sqrt{3}}$ $\quad -180^0 \le \theta < 180^0$

Give exact values.

$\tan\theta_R = +\frac{1}{\sqrt{3}}$



$\theta_R = 30^\circ$



Q2: $\theta = 180^\circ - 30^\circ$

$\theta = 150^\circ$

Q4: $\theta = 0^\circ - 30^\circ$

$\theta = -30^\circ$

Which quadrants will the answers be in?

Q2 and Q4

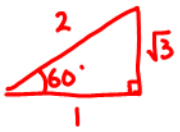
How many answers will there be?

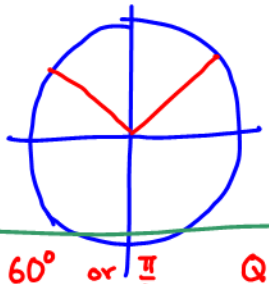
2 solutions in 1 rotation.

d) $\csc\theta = \frac{2}{\sqrt{3}}$ $\quad -3\pi \le \theta < \pi$ $\quad -540° \quad 180°$

Give exact values.

$\csc\theta_R = +\frac{2}{\sqrt{3}}$





Which quadrants will the answers be in?

Q1 and Q2.

How many answers will there be?

2 rotations so

4 possible solutions

Q1: $\theta = 60°$ or $\frac{\pi}{3}$ $\quad$ Q1: $\theta = -300°$ or $-\frac{5\pi}{3}$

Q2: $\theta = 120°$ or $\frac{2\pi}{3}$ $\quad$ Q2: $\theta = -240°$ or $-\frac{4\pi}{3}$

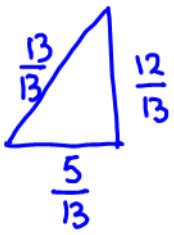
Example 2. The point $Q(5,-12)$ lies on the terminal arm of an angle θ. Determine the exact value of each of the trigonometric ratios and the smallest positive value of θ in radians.

Is the point $Q(5,-12)$ on the unit circle? Explain.

not on unit circle $\quad 5^2 + (-12)^2 \neq 1$

radius = $\quad r^2 = 5^2 + (-12)^2 \quad \boxed{r = 13}$

Unit circle approach



(5,-12) not on unit circle.

$\left(\frac{5}{13}, \frac{-12}{13}\right)$ is on the unit circle

$\sin\theta = \frac{-12}{13}$ $\quad \csc\theta = -\frac{13}{12}$

$\cos\theta = \frac{5}{13}$ $\quad \sec\theta = \frac{13}{5}$

$\tan\theta = \frac{\frac{-12}{13}}{\frac{5}{13}} = \frac{-12}{5}$ $\quad \cot\theta = -\frac{5}{12}$

Non unit circle approach

$\sin\theta = \frac{y}{r} = \frac{-12}{13}$ $\quad \csc\theta = -\frac{13}{12}$

$\cos\theta = \frac{x}{r} = \frac{5}{13}$ $\quad \sec\theta = \frac{13}{5}$

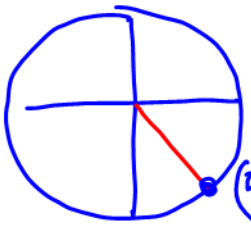
$\tan\theta = \frac{y}{x} = \frac{-12}{5}$ $\quad \cot\theta = -\frac{5}{12}$

finding θ is the same as

$\cos\theta_R = \frac{5}{13}$

$\theta_R = \cos^{-1}\left(\frac{5}{13}\right)$

$\theta_R = 1.1760$



~~Q1: $\theta = 1.1760$~~

Q4: $\theta = 2\pi - 1.1760$

$= 5.1040$

HW p201 #10-12, 15, 16, C1, C2

*20, 22 C4.

**Example 3. When you keystroke $\sin^{-1}\left(\frac{1}{2}\right)$ on your calculator, is your calculator giving you "the" angle whose sine is $\frac{1}{2}$ or "an" angle whose sine is $\frac{1}{2}$?

When you keystroke $\sin^{-1}\left(-\frac{1}{2}\right)$ on your calculator, what are you asking your calculator to do? Why do you think your calculator gives you the answer it does?

What is $\sin\left(\sin^{-1}\left(\frac{1}{2}\right)\right)$?

What is $\sin\left(\sin^{-1}\left(-\frac{1}{2}\right)\right)$?

What is $\sin^{-1}\left(\sin\left(\frac{\pi}{6}\right)\right)$?

What is $\sin^{-1}\left(\sin\left(\frac{5\pi}{6}\right)\right)$?

What is $\sin^{-1}\left(\sin\left(\frac{11\pi}{6}\right)\right)$?