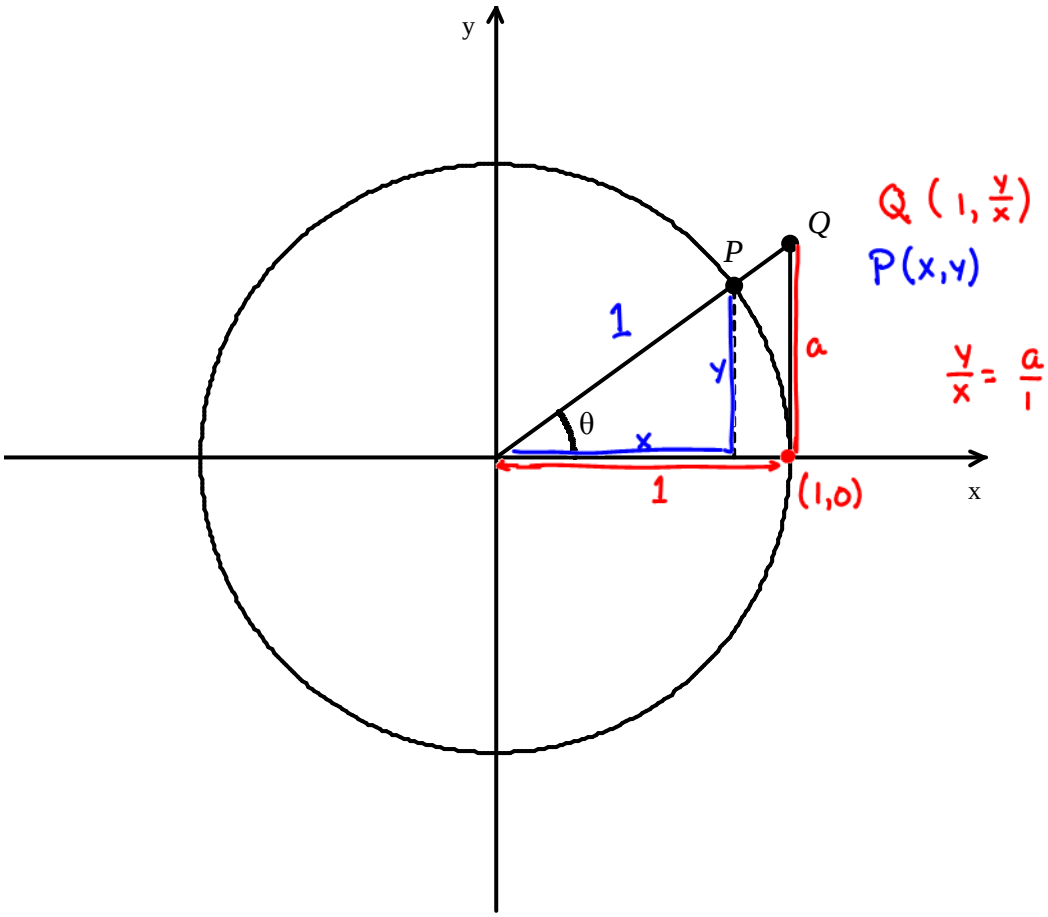
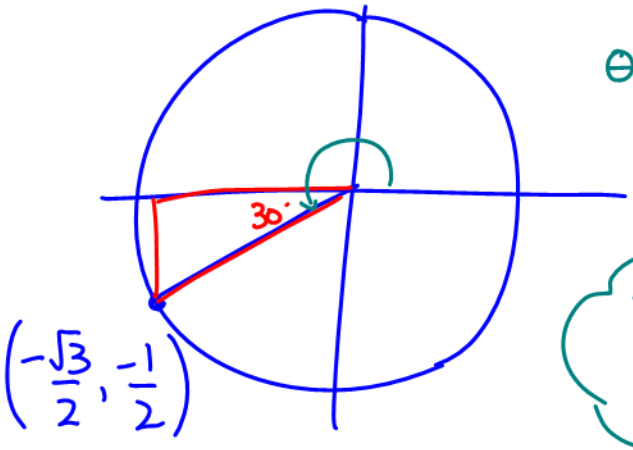
4.3A Warmup

1. Given the unit circle below, determine the coordinates of points *P* and *Q* in terms of θ.



2. Determine such θ that $P(\theta) = \left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$ with $0 \le \theta < 6\pi$



$\theta = 210°$ or $\theta = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$

$\frac{7\pi}{6}, \frac{19\pi}{6}, \frac{31\pi}{6}$

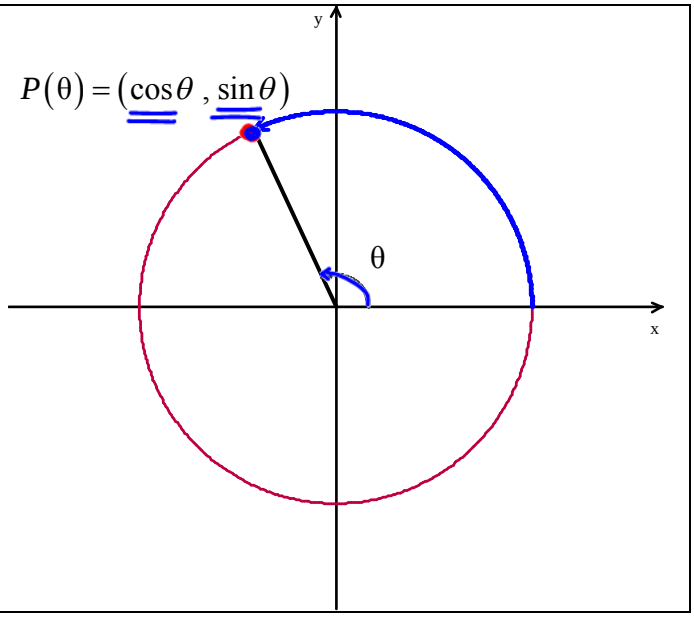
4.3A The Trigonometric Ratios

Recall the definitions of sine and cosine for rotation angles. On the unit circle, what do these ratios describe?

$\cos\theta = \frac{x}{r} = \frac{x}{1} = x$

$\sin\theta = \frac{y}{r} = \frac{y}{1} = y$

This means that the coordinates of any point on the unit circle can be described in terms of the rotation angle as $(\cos\theta, \sin\theta)$



Because $\tan\theta = \frac{y}{x}$ (y = $\sin\theta$, x = $\cos\theta$), we can then also say that $\tan\theta = \frac{\sin\theta}{\cos\theta}$ (provided $\cos\theta \neq 0$)

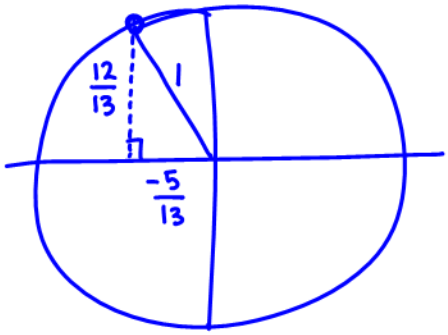
Reciprocal Trigonometric Ratios

The reciprocals of the trigonometric ratios occur often, and they are given special names.

Secant	Cosecant	Cotangent
$\sec\theta = \frac{1}{\cos\theta}$, $\cos\theta \neq 0$	$\csc\theta = \frac{1}{\sin\theta}$, $\sin\theta \neq 0$	$\cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$, $\sin\theta \neq 0$

Example 1. The point $\left(-\frac{5}{13}, \frac{12}{13}\right)$ lies on the terminal arm of of an angle θ on the unit circle.

a) Draw a diagram to illustrate this information.



b) Determine the values of the six trigonometric ratios for θ

$\sin\theta = y = \frac{12}{13}$ $\quad \csc\theta = \frac{13}{12}$

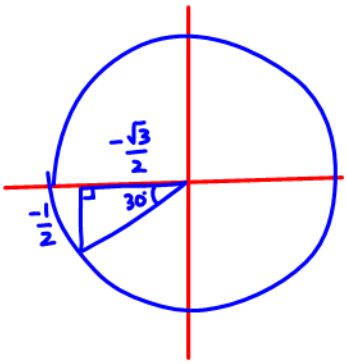
$\cos\theta = -\frac{5}{13}$ $\quad \sec\theta = -\frac{13}{5}$

$\tan\theta = \frac{\frac{12}{13}}{-\frac{5}{13}} = -\frac{12}{5}$ $\quad \cot\theta = -\frac{5}{12}$

special triangle or a quadrant angle.

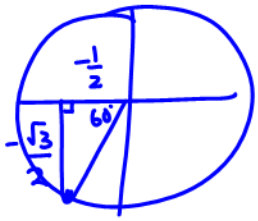
Example 2. Determine the exact value for each of the following. Include a diagram for each question.

a) $\sin\left(\frac{7\pi}{6}\right)$



$\sin\left(\frac{7\pi}{6}\right) = -\frac{1}{2}$

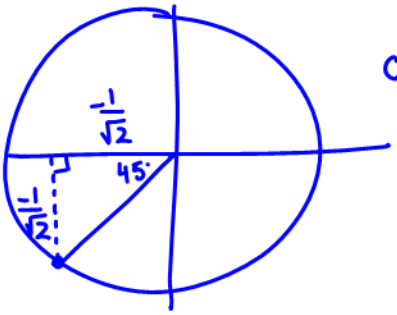
b) $\cos\left(-\frac{2\pi}{3}\right) = x$



$\cos\left(-\frac{2\pi}{3}\right) = -\frac{1}{2}$

c) $\sec 225^0$

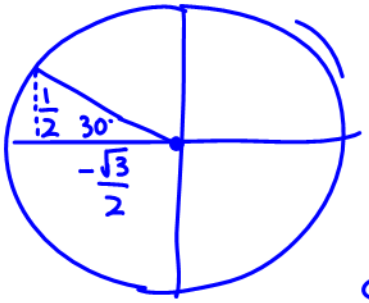
find $\cos 225°$ first.



$\cos 225° = -\frac{1}{\sqrt{2}}$

$\sec 225° = -\sqrt{2}$

d) $\cot\left(+150^0\right)$

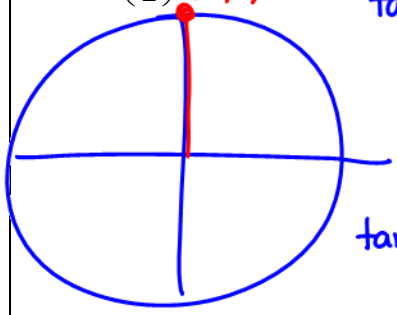


$\cot\theta = \frac{\cos\theta}{\sin\theta}$

$= \frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}}$

$\cot\theta = -\sqrt{3}$

e) $\tan\left(\frac{\pi}{2}\right)$ (0,1)



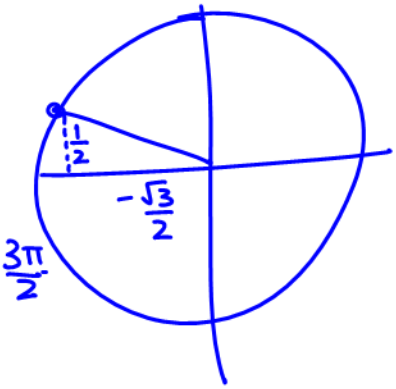
$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{y}{x}$

$= \frac{1}{0}$ not possible.

$\tan\theta$ is undefined at $\theta = \frac{\pi}{2}$ and $\theta = \frac{3\pi}{2}$

*quadrant angles are on the border of 2 quadrants.

f) $\csc\left(+\frac{5\pi}{6}\right)$



$\csc\theta = \frac{1}{\sin\theta}$

$\sin\theta = \frac{1}{2}$

$\csc\theta = 2$

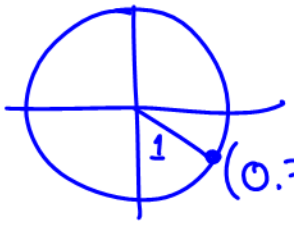
allowed to use a calculator.

Example 3. Determine the approximate value for each trigonometric ratio. Round your answers to four decimal places. What does each answer represent on the unit circle?

a) $\sin 3.2$ $= -0.05837$

this is the y-coordinate of the point on the terminal arm where it meets the unit circle.

b) $\cos 320^0 = 0.76604$



i) y value can be found from
a) sine on calc.
b) pythagorean th.

c) $\sec 139^0$

$\sec\theta = \frac{1}{\cos\theta}$

$\cos 139^\circ = -.75471$

$\sec 139^\circ = \frac{1}{-.75471} = -1.3250$

d) $\csc \frac{\pi}{5}$

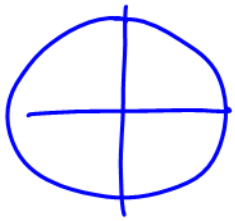
$\sin \frac{\pi}{5} = .58779$

$\csc \frac{\pi}{5} = \frac{1}{.58779} = 1.7013$

Example 4. What are the the largest and smallest values of

a) $\cos\theta$? $|\cos\theta| \le 1$

$0 \le |\cos\theta|$



b) $\sin\theta$?

$0 \le |\sin\theta| \le 1$

c) $\tan\theta$?

$0 \le |\tan\theta|$

$\tan\theta \in \mathbb{R}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

d) $\sec\theta$?

$1 \le |\sec\theta| < \infty$

$1 \le \sec\theta < \infty$

or

$-1 \ge \sec\theta > -\infty$

$\sec\theta = \frac{1}{\cos\theta}$

p201 #1-9, 13, 14, 17

Quiz next Wednesday.