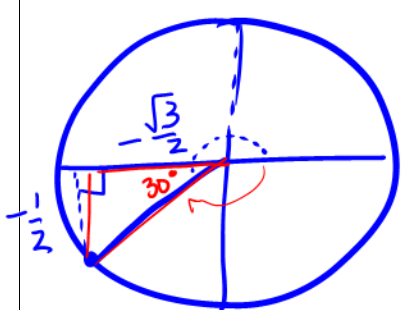
4.2B The Unit Circle – Given the point, finding the angle

The function $P(\theta)=(x,y)$ gives the coordinates of the point on the unit circle associated with a nagle of rotation (or arclength) of θ. Thus $P\left(-\frac{3\pi}{4}\right)=\left(\frac{-1}{\sqrt{2}},\frac{-1}{\sqrt{2}}\right)$ relates the point $\left(\frac{-1}{\sqrt{2}},\frac{-1}{\sqrt{2}}\right)$ with the angle (or arclength) of $-\frac{3\pi}{4}$.

Example 1. Identify a measure for the central angle θ in the interval $0\le\theta<2\pi$ such that $P(\theta)$ is the given point:

a) $\left(-\frac{\sqrt{3}}{2},-\frac{1}{2}\right)$

Draw the unit circle and label the diagram in an appropriate way. Use your knowledge of special triangles to determine the value of θ.



$\theta = 180^\circ + 30^\circ$
$= 210^\circ$

$\theta = \pi + \frac{\pi}{6}$

$\theta = \frac{7\pi}{6}$

This question is really asking what value of θ makes $P(\theta) = \left(-\frac{\sqrt{3}}{2},-\frac{1}{2}\right)$? Or what rotation angle brings you to the point $\left(-\frac{\sqrt{3}}{2},-\frac{1}{2}\right)$?

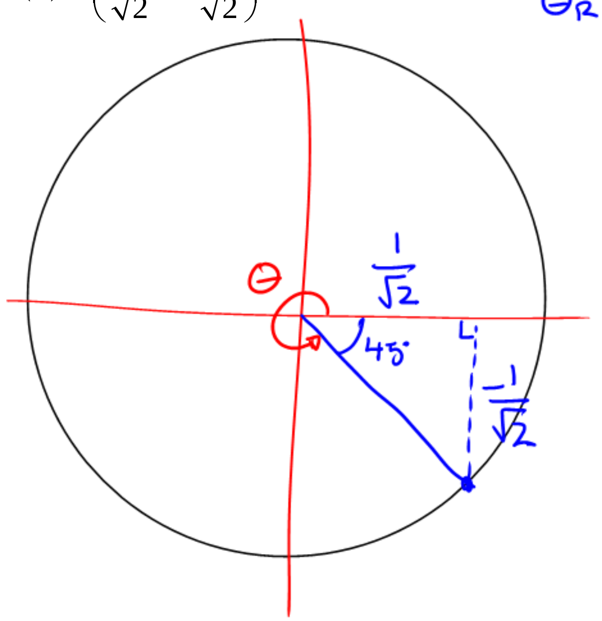
How many answers does this question have?

infinite because we can consider full rotations

Why has the interval been limited to $0\le\theta<2\pi$?

- you find only 1 answer and the rest are found by adding $n2\pi$ to it.

b) $P(\theta)=\left(\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}}\right)$



$\theta_R = 45^\circ$ or $\frac{\pi}{4}$

$\theta = 360^\circ - 45^\circ$
$= 315^\circ$

or $\theta = 2\pi - \frac{\pi}{4}$
$= \frac{7\pi}{4}$

no restriction answer

$\theta = 315^\circ + n(360^\circ)$

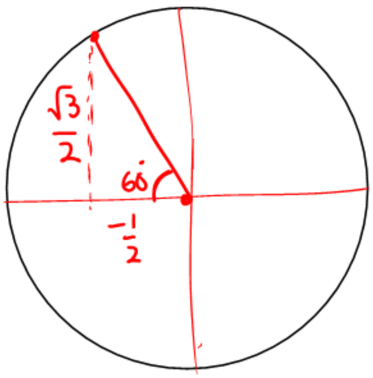
or

$\theta = \frac{7\pi}{4} + n(2\pi)$

n is any integer

6π or 3 full rotations apart

Example 2. Determine all values for θ in the interval $-\pi \le \theta < 5\pi$ such that $P(\theta) = \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$



$\theta = 180 - 60 = 120°$ or $\theta = \pi - \frac{\pi}{3}$

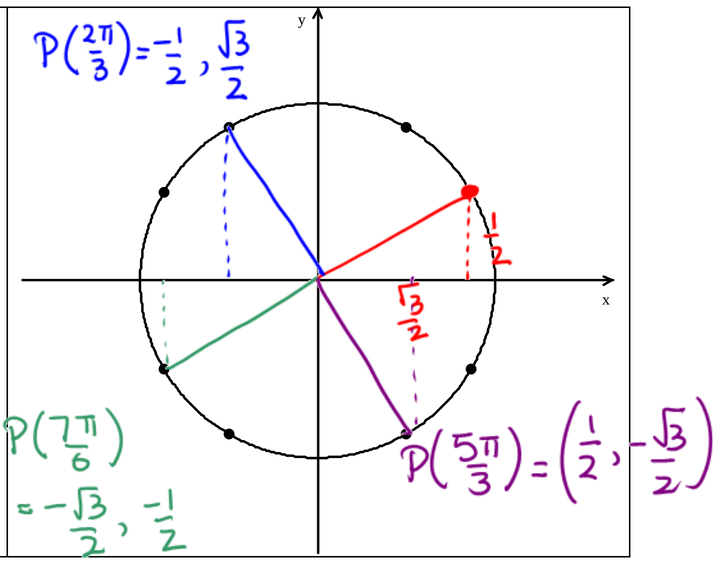
$= \frac{2\pi}{3}$

$\frac{8\pi}{3}$, $\frac{14\pi}{3}$

$\theta = \frac{2\pi}{3}, \frac{8\pi}{3}, \frac{14\pi}{3}$

Example 3. What is the relationship between the points that are $\frac{1}{4}$ rotation apart on the unit circle?

Start with the point $P\left(\frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$. Move $\frac{1}{4}$ rotation from this point. Determine this new point and its coordinates. Repeat this process by moving $-\frac{1}{4}$ rotation from the original point.



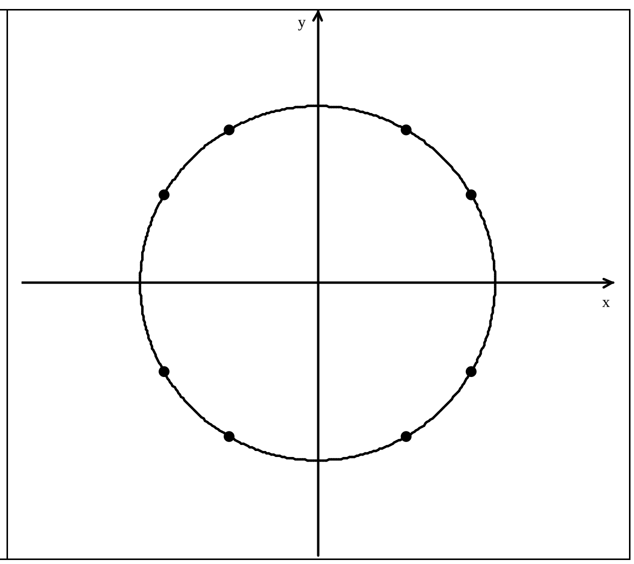
Repeat these steps with the point $P\left(\frac{\pi}{3}\right) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$P\left(\frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$P\left(\frac{\pi}{6} + \frac{\pi}{2}\right) = \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$P\left(\frac{\pi}{6} + 2\frac{\pi}{2}\right) = \left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

$P\left(\frac{\pi}{6} + \frac{3\pi}{2}\right) = \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$



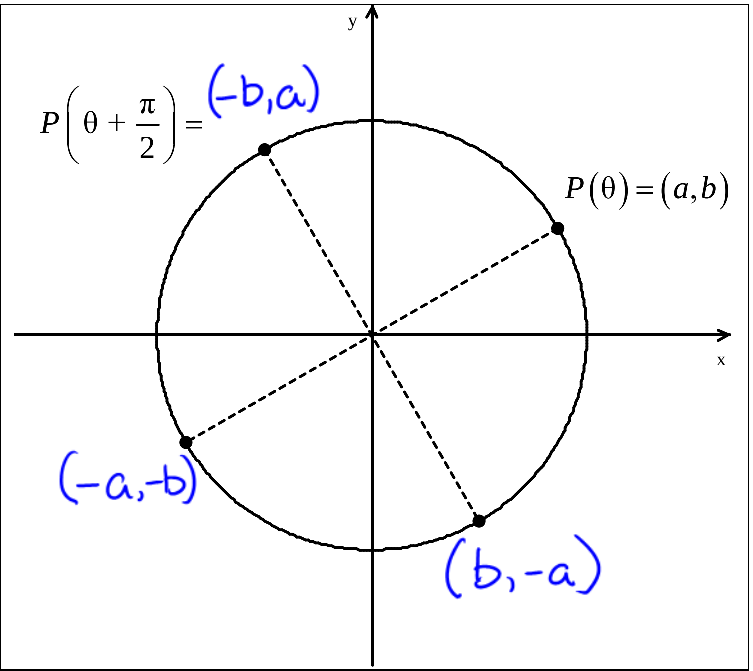
Summarize your findings in the diagram to the right.

$P(\theta) = (a, b)$

$P(\theta + \frac{\pi}{2}) = (-b, a)$

$P(\theta + \frac{2\pi}{2}) = (-a, -b)$

$P(\theta + \frac{3\pi}{2}) = (b, -a)$



Last class = 4.2A p187 #1-4, 10, 11

4.2b p 187 # 5-7, 9, 12, 13, 15, C1, C2.