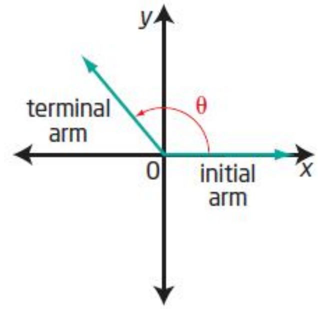
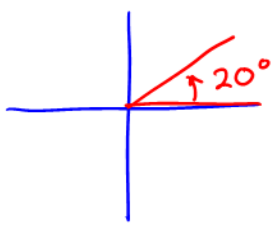
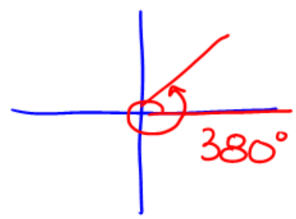
4.1B Coterminal Angles

Recall that an angle θ is in ***standard position*** in the coordinate plane if the following two requirements are satisfied:

a) the vertex of the angle is at the origin b) the initial arm is the positive x-axis Also by convention, a counter clockwise rotation is a positive angle and a clockwise rotation is a negative angle.	

Draw angles of 20^0 and 380^0 in standard position. What is different about these two angles? What is the same about these two angles?

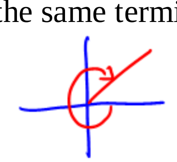




same: final position

different: 380° had a full rotation counterclockwise first.

What negative angle would share the same terminal arm?

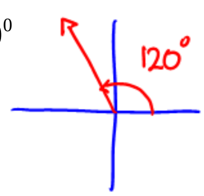
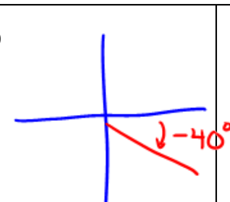
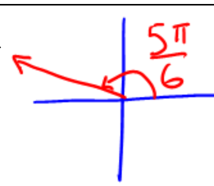
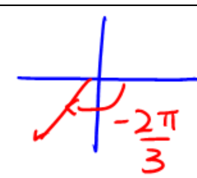


$-340°$

Coterminal Angles

Coterminal angles are standard position angles that share a common terminal arm. Thus 20^0, 380^0 and -340^0 are coterminal angles.

1. Determine one negative and one positive angle that is coterminal with each of the following angles:

a) 120^0 	b) -40^0 	c) $\frac{5\pi}{6}$ 	d) $-\frac{2\pi}{3}$ 
$480°$ $-240°$	$320°$ $-400°$	$\frac{5\pi}{6} + 2\pi = \frac{17\pi}{6}$ $\frac{5\pi}{6} - 2\pi = -\frac{7\pi}{6}$	$-\frac{2\pi}{3} + 2\pi = \frac{4\pi}{3}$ $-\frac{2\pi}{3} - 2\pi = -\frac{8\pi}{3}$

coterminal angles can be found by $\pm 360°$ (full rotations)

Every angle in standard position has an infinite number of coterminal angles associated with it. For example, consider the standard position angle $\theta = 135^0$. How could you quickly determine two positive and two negative angles that are coterminal with θ?

$\theta_1 = 135^\circ + 360^\circ$ $\qquad$ $\theta_{-1} = 135^\circ + -1(360^\circ)$

$\theta_2 = 135^\circ + 2(360^\circ)$ $\qquad$ $\theta_{-2} = 135^\circ + -2(360^\circ)$

In general, we can say that the angles coterminal with 135° are $135^0 \pm (360^0)n$, where n is any natural number. We could also express this as $\boxed{135^0 + (360^0)n\text{, where } n \text{ is any integer.}}$

Similarly, two positive and two negative angles coterminal with $\theta = \frac{3\pi}{4}$ are

$\theta_1 = \frac{3\pi}{4} + 2\pi$ $\qquad$ $\theta_{-1} = \frac{3\pi}{4} + -2\pi$

$\theta_2 = \frac{3\pi}{4} + 2(2\pi)$ $\qquad$ $\theta_{-2} = \frac{3\pi}{4} + -2(2\pi)$

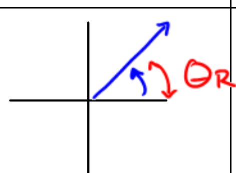
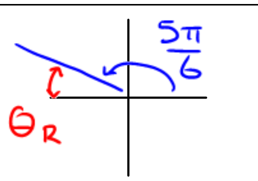
In general, the angles coterminal with $\frac{3\pi}{4}$ are $\frac{3\pi}{4} + 2\pi n$, where n is any integer.

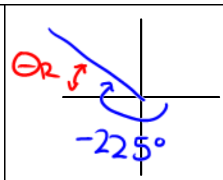
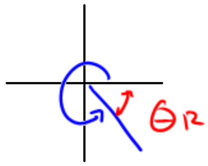
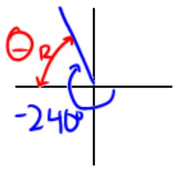
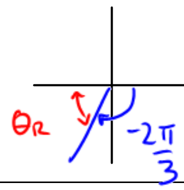
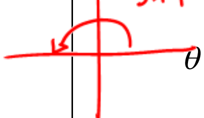
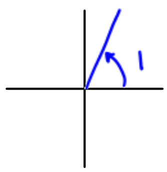
Generalization:
In general, consider an angle θ in standard position. Coterminal angles of θ will have the form

$\boxed{\theta + n360^\circ}$ or $\boxed{\theta + n2\pi}$ where n is any integer.

2. For each angle in the table below, draw the angle, state the quadrant it terminates in, give a positive and negative coterminal angle and state the reference angle.

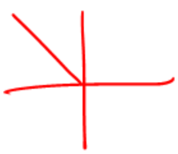
reference : size of angle made to nearest x-axis.

Angle	Drawing	Quadrant	Two coterminal angles	Reference angle
$\theta = 45^\circ$		1	405° -315°	45°
$\theta = \frac{5\pi}{6}$		2	$\frac{17}{6}\pi$ $-\frac{7}{6}\pi$	$\frac{\pi}{6}$

$\theta = -225^{\circ}$		2	135° -585°	$\theta_R = 45^{\circ}$
$\theta = \frac{5\pi}{3}$		4	$\frac{11\pi}{3}$ $-\frac{\pi}{3}$	$\theta_R = \frac{\pi}{3}$
$\theta = -240^{\circ}$		2	120° -600°	$\theta_R = 60^{\circ}$
$\theta = -\frac{2\pi}{3}$		3	$\frac{4\pi}{3}$ $-\frac{8\pi}{3}$	$\theta_R = \frac{\pi}{3}$
$3.14^R \doteq 3^R$  $\theta = 1$		1	$1 + 2\pi$ $1 - 2\pi$	$\theta_R = 1$

3. Find the smallest positive angle coterminal with

a) -5000^{0} $+ 14(360^{\circ})$



$-5000 + 5040$

40°

b) $\frac{88\pi}{3}$

$\frac{88}{3}\pi - 14(2\pi)$

$\frac{88\pi}{3} - 28\pi$

$\frac{88\pi}{3} - \frac{84\pi}{3} = \frac{4\pi}{3}$

4. Do the expressions $\theta = \frac{5\pi}{6} + 2\pi n,\ n \in I$ and $\theta = -\frac{19\pi}{6} + 2\pi n,\ n \in I$ represent the same set of angles? Explain.

"are these coterminal?"

$\frac{5\pi}{6} - 2\pi = -\frac{7\pi}{6}$

$-\frac{7\pi}{6} - 2\pi = -\frac{19\pi}{6}$

$\frac{5\pi}{6}$ and $-\frac{19\pi}{6}$ are coterminal.

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