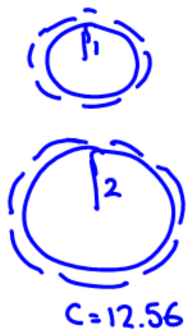
Radian Measure

Developing Radian Measure

1. What is the circumference of a circle with radius ‘*r*’ ? $C = 2 \cdot \pi \cdot r$

2. How many degrees are there in a complete rotation? $360°$

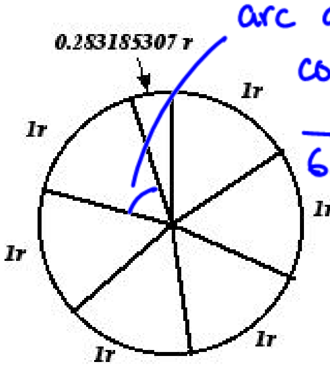
Fill in the following table:



Radius	Circumference	Number of times the radius “fits around the circumference”
1	$2 \cdot \pi$	6.28 $= 2\pi \div 1$
2	4π	6.28 : $4\pi \div 2$
3	6π	6.28 : $6\pi \div 3$
r	$2\pi r$	6.28 $2\pi r \div r$

What can you conclude about the relationship between the radius and the number of times the radius can wrap around the circle? it always takes 6.28 radiuses (2π) to wrap around the circumference.

Graphically, the radius will wrap around the circle as follows. (The radius is *r*.)



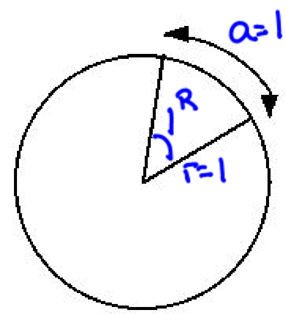
arc always covers $\frac{1}{6.28}$ or $\frac{1}{2\pi}$ of the circle

If the radius varies in size, will the corresponding central angle measurements change or stay the same?

stays the same.

Radians are related to the radius. According to the diagram which follows, we can see the relationship between the radius, arc length, and 1 radian.

An arc of length ____1____ of a circle of radius ____1____subtends an angle of ____1____ radian.



an arc of length "r" in a circle of radius "r" has an angle of 1 radian.

In other words, 1 radian is the measure of a central angle that is subtended by an arc equal in length to the ____radius.____.

$$\# \text{radians} = \frac{\text{arc length}}{\text{radius}}$$

Developing the Relationship Between Radians and Degrees

1. How many degrees does it take to "go around a circle?" 360°

2. How many times does the radius "go around a circle?" 2π

Both measurements can take us completely around the circle. Complete the following relationships:

Going around the circle in degrees = Going around the circle in radians

$\boxed{360^\circ}$ $\boxed{2\pi^R}$

* $\pi^R = 180^\circ$

Converting Radians to Degrees

If 2π radians $= 360^\circ$, then

$$1 \text{ radian} = \left[\frac{360}{2\pi}\right]^\circ = \left[\frac{180}{\pi}\right]^\circ$$

In general,

Converting Degrees to Radians

If $360^\circ = 2\pi$ radians, then

$$1^\circ = \left[\frac{2\pi}{360}\right] \text{radians} = \left[\frac{\pi}{180}\right] \text{radians}$$

In general,

Express the following in degrees and radians:

	Degrees	Radians
a) A half rotation	180°	π
b) Two complete rotations	720°	4π
c) A quarter rotation	90°	$\pi/2$
d) One third of a rotation	120°	$2\pi/3$
e) One sixth of a rotation	60°	$\pi/3$

Note: Any angle measurement given without a unit is assumed to be in radians.
E.g. $\theta = 2$ means $\theta = 2$ radians .

Convert from radians to degrees.

eg $\frac{\pi}{3}$ substitute $\pi = 180°$

$\frac{(180°)}{3} = 60°$

$\frac{3\pi}{4}$ $\frac{3 \times (180°)}{4} = 135°$

$\frac{1}{2}\pi$ $\frac{1}{2}(180°) = 90°$

2^R

$\frac{2}{2\pi} = \frac{\square}{360°}$

cross multiply and divide.

$2^R \doteq 114.6°$

Convert degrees to radians

$\pi^R = 180°$

make a fraction involving 180°

eg 120° $\frac{120°}{180°} = \frac{2}{3}$

120° is $\frac{2}{3}$ of 180° $= \frac{2}{3}\pi$ or $\frac{2\pi}{3}$

or eg 71° $\frac{1.24}{2\pi} = \frac{71°}{360°}$ $71° \doteq 1.24^R$

$\pi = 180°$

$\frac{\pi}{2} = 90°$

$\frac{\pi}{3} = 60°$

$\frac{\pi}{4} = 45°$

$\frac{\pi}{6} = 30°$

Convert each of the following to radians or degrees.

(a) $30^{\circ} = \square$ $\frac{\pi}{6}$

(b) $\frac{\pi}{2} = \square$ 90°

(c) $\frac{2\pi}{3} = \square$ 120°

(d) $36^{\circ} = \square$ $\frac{\pi}{5}$

$\frac{36}{180}$

(e) $225^{\circ} = \square$ $\frac{5\pi}{4}$

(f) $-\frac{\pi}{4} = \square$ -45°

(g) 2 radians $\doteq \square$ 114.6°

$\frac{2}{2\pi} = \frac{\quad}{360^{\circ}}$

(h) $252^{\circ} = \square$ $\frac{7\pi}{5}$

(i) $\frac{7\pi}{6} = \square$ 210°

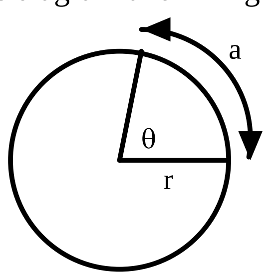
(j) $45^{\circ} = \square$ $\frac{\pi}{4}$

(k) $\frac{3\pi}{5} = \square$ 108°

(l) 7 radians $\doteq \square$ 401°

Finding arc length

Write the proportion by following the diagram and filling in the blanks.



radians = # of radiuses that fit around the circumference

$\theta^{R} = \frac{a}{r}$

Note: The angle □ is measured in **radians.**

$$\frac{\text{arc length}}{\text{total circumference}} = \frac{\text{central angle measure}}{\text{total angle measure of 1 rotation}}$$

$$\frac{\boxed{a}}{\boxed{2\pi r}} = \frac{\boxed{\theta}}{\boxed{2\pi}}$$

By solving for '*a*' in the relationship, the equation for calculating the arc length is:

$a = \boxed{\theta \cdot r}$ (where the angle θ is in radians) $a = r\theta$

If we set up the same relationship but with the central angle in degrees, we get:

$$\frac{\text{arc length}}{\text{total circumference}} = \frac{\text{central angle measure}}{\text{total angle measure of 1 rotation}}$$

$$\frac{\boxed{a}}{\boxed{2\pi r}} = \frac{\boxed{\theta}}{\boxed{360°}}$$

$$a = \boxed{\frac{2\pi r\theta}{360°}}$$ (where the angle θ is in degrees)

Note how much simpler the arc length formula is if we use *radian* measure for the central angle.

Example 1:

Determine the arc length in a circle of radius 10 cm if:

(a) the central angle is 5 radians

$a = r\theta$

$= 10\text{cm}\,(5)$

$a = 50\text{cm}.$

b) the central angle is 25º.

$$a = \frac{2\pi r\theta}{360°} = \frac{2\pi(10)(25°)}{360°}$$

$$= 4.36 \text{ cm}$$

Example 2:

Determine the central angle (in radians) subtended by an arc of length 3 cm in a circle of radius 10cm .

$a = r\theta$

$3 = 10\theta$

$\theta = \frac{3}{10}$ or 0.3^R

p175 #1-6,
12-15, 17, 19.