p133 #5,8,11,13 C2 C3

This means that in testing for factors, we might also incorporate the factors of the leading term and look also at substituting $\dfrac{\pm \textit{factors of constant}}{\textit{factors of leading coeff}}$

Warmup 3.4a

Without using a the graphical features of a graphing calculator, determine the following characteristics of the polynomial function $f(x) = -x^3 + 2x^2 + 5x - 6$

- the degree of the polynomial 3
- the sign of the leading coefficient $-$
- the zeros of the function

  $P(-2)=0$

  $P(1)=0$

  $P(3)=0$

- the factored form of the function

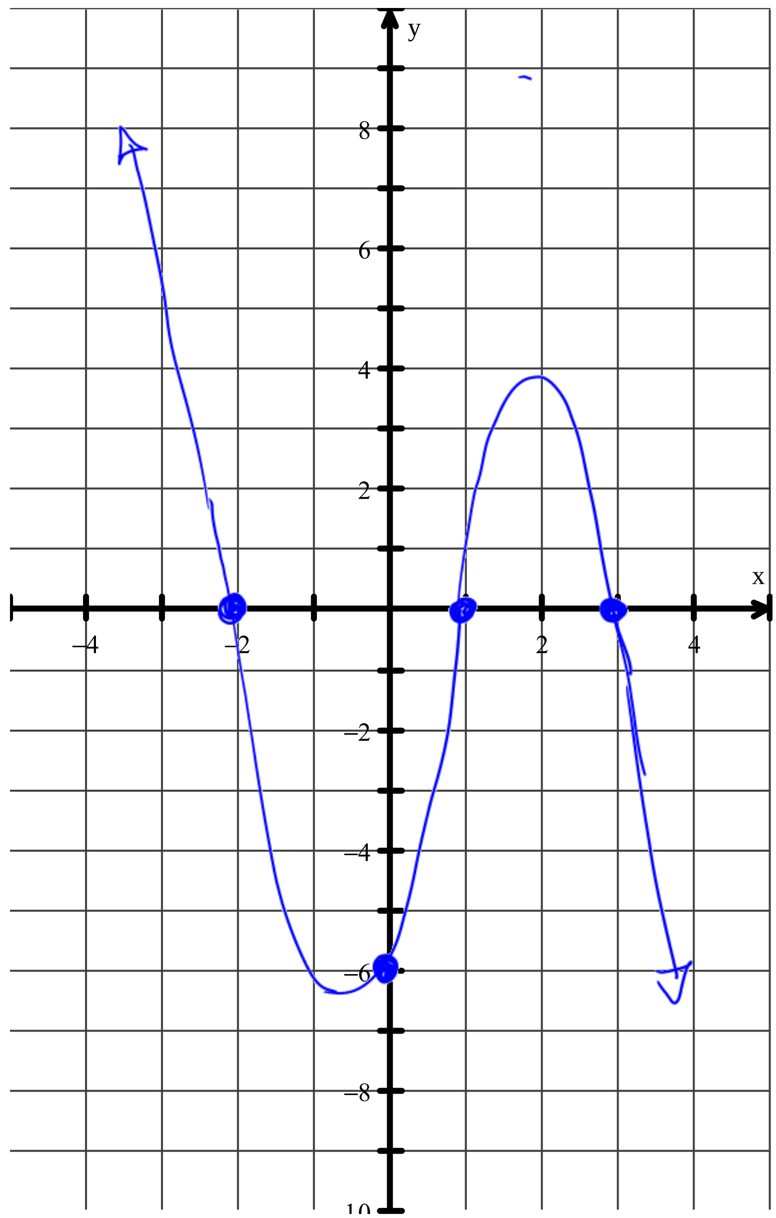
  $-1(x+2)(x-1)(x-3)$

- the *y*-intercept -6
- the interval(s) where the function is positiv

  $(-\infty,-2]$ $(1,3)$

- the interval(s) where the function is negative

  $(-2,1)$ $(3,\infty)$

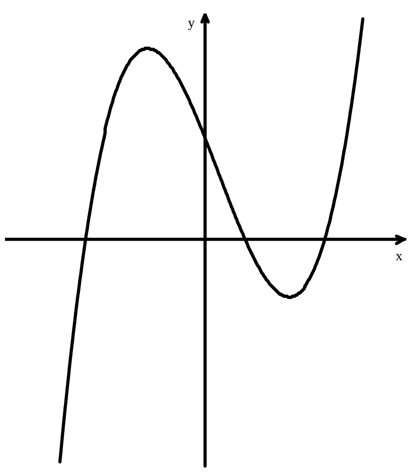


3.4a Equations and Graphs of Polynomial Functions

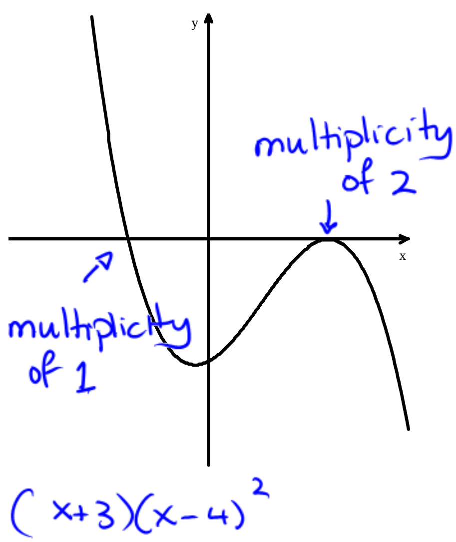
$x-3=0 \longrightarrow x=3$

The zeros of any polynomial function $y = f(x)$ are also the *x*-intercepts, and correspond to the roots of the equation $f(x)=0$. When a polynomial is in factored form, the zeros can be found by equating each of the factors to zero. The function $f(x)=(x-3)^2(x+2)$ has two identical zeros at $x=3$ and a third zero at $x=-2$. These are also the roots of the equation $(x-3)(x-3)(x+2)=0$. The **multiplicity** (or order) of a zero is the number of times that its corresponding factor is repeated. Thus for the function above, there is a zero of multiplicity 2 at $x=3$ and a zero of multiplicity of 1 at $x=-2$.

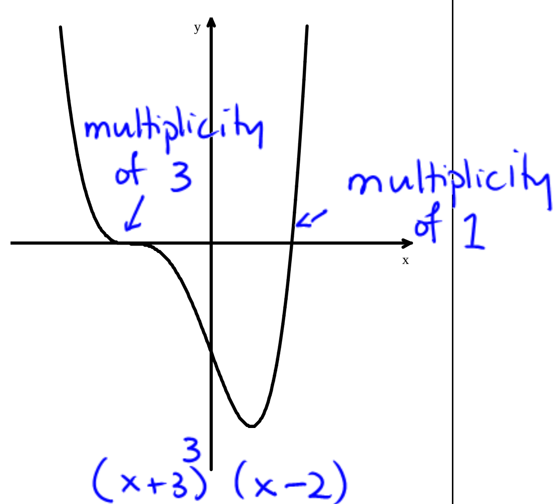
The shape of a graph near its zero depends on its multiplicity.



All three zeros have a multiplicity of 1 – the graph behaves like a line at these zeros.



The zero on the right has a multiplicity of 2 – the graph behaves like a parabola at this zero.

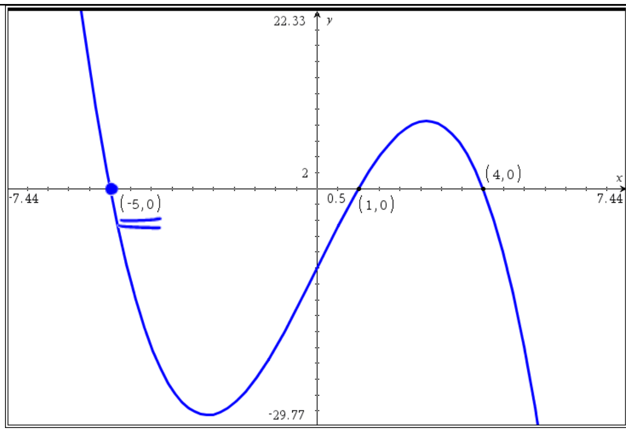


The zero on the left has a multiplicity of 3 – the graph behaves like $y = x^3$ at this zero.

When a graph reaches a zero with odd multiplicity, the sign of the function changes. When a graph reaches a zero with even multiplicity, the sign of the function does not change.

Example 1. For each graph, determine

a) the least possible degree
b) the sign of the leading coefficient
c) the x-intercepts and the factors of the function with the least possible degree
d) the intervals where the function is positive and negative



degree: 3

L.C = negative

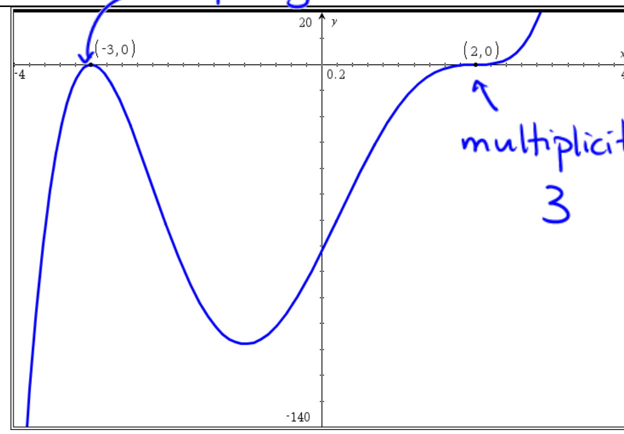
x-int: $(-5,0)$, $(1,0)$, $(4,0)$

$y = -(x+5)(x-1)(x-4)$

positive: $(-\infty, -5)$ & $(1,4)$

negative: $(-5,1)$ & $(4,\infty)$

multiplicity 2



multiplicity 3

degree: 5, LC: +

x-int: $(-3,0)$, $(2,0)$

$y = +(x+3)^2(x-2)^3$

positive: $(2,\infty)$

$(-\infty,-3)$ & $(-3,2)$

Example 2. Sketch a graph for each of the following polynomial functions.

a) $f(x) = -2(x-3)(x+1)(x+4)$

Leading term -2 degree $= 3$

End behaviour Q2 → Q4

Zeros $-4, -1, 3$

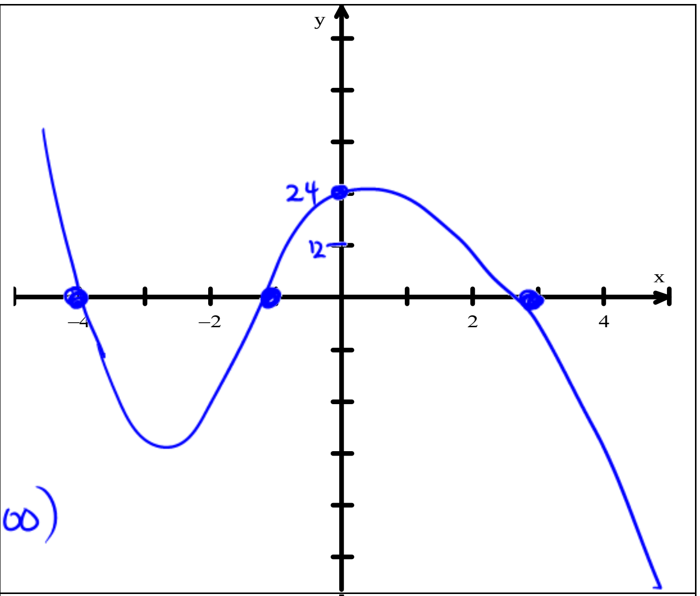
y-intercept 24

Intervals where the function is positive

and negative

$(-\infty, -4)$ or $(-1, 3)$

$(-4, -1)$ $(3, \infty)$



b) $f(x) = -(x)^3(x-2)^2(x+3)$

Leading term negative degree : 6

End behaviour Q3 → Q4

Zeros $0, 2, -3$

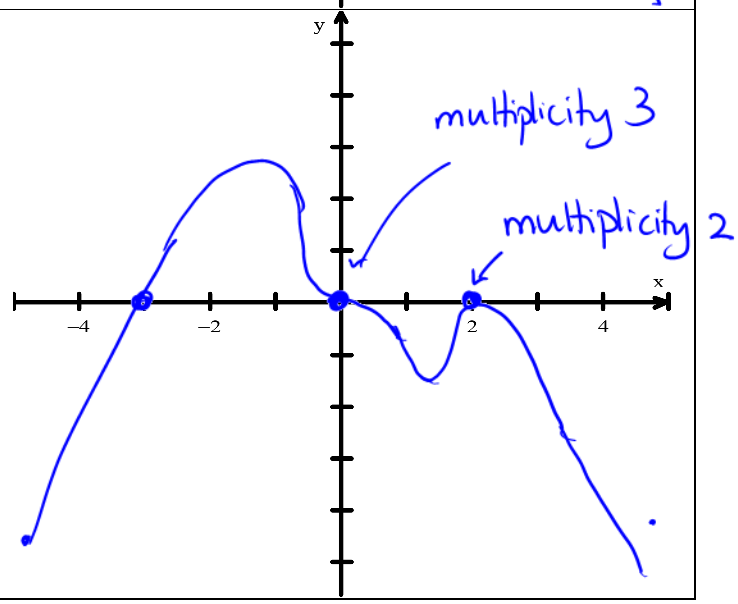
y-intercept 0

Intervals where the function is positive

and negative

$(-3, 0)$

$(-\infty, -3)$ $(0, 2), (2, \infty)$



c) $f(x) = 2x^4 - 18x^2$

$2x^2(x^2 - 9)$

$2x^2(x+3)(x-3)$

Leading term (+)

End behaviour Q2 → Q1

degree: even

Zeros $-3, 0, 3$

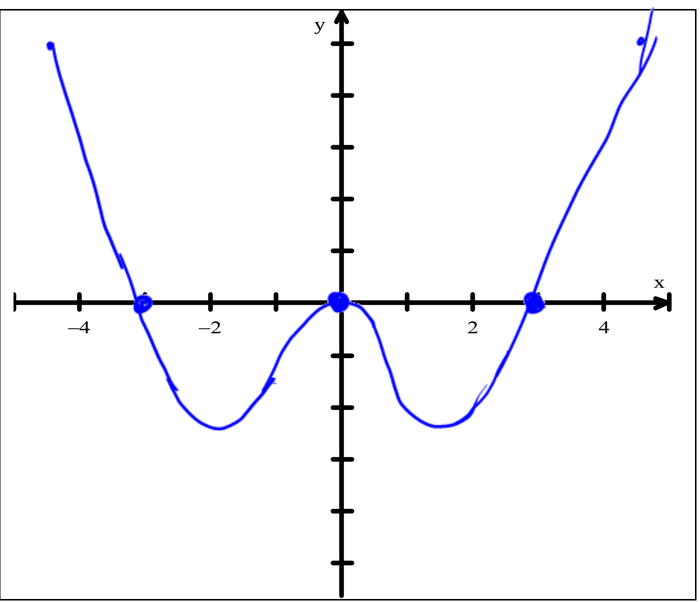
y-intercept 0

Intervals where the function is positive

and negative

$(-\infty, -3)$ $(3, \infty)$

$(-3, 0)$ $(0, 3)$



Example 3. Determine the equation of a cubic function with zeros of -2, 3 and 5 and with a y-intercept of -30.

$y = k(x+2)(x-3)(x-5) \longleftarrow$ when $x=0$, $y=-30$

$-30 = k(0+2)(0-3)(0-5)$

$-30 = k(30)$

$k = -1$

$\boxed{\therefore\ f(x) = -1(x+2)(x-3)(x-5)}$

Example 4. A quartic function has zeros of 2 (multiplicity of two), -1 and -6. If the function passes through the point $\boxed{(1,-28)}$, what must its y-intercept be?

$y = k(x-2)^2(x+1)(x+6)$

$-28 = k(1-2)^2(1+1)(1+6)$

$-28 = 14k$

$k = -2$

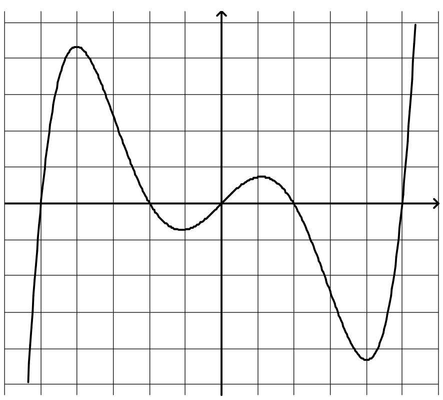
$f(x) = -2(x-2)^2(x+1)(x+6)$

$f(0) = -48$

$\boxed{y\text{-int} = -48}$

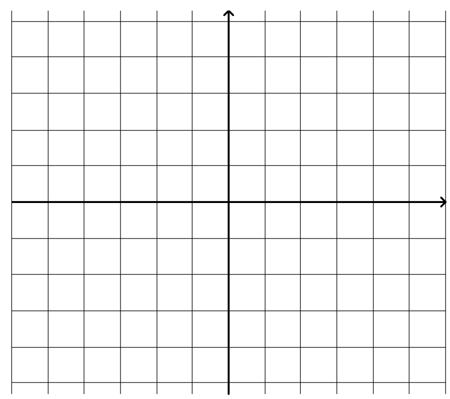
Warmup 3.4B

1. What could the degree of the following polynomial function be?



2. Determine the zeros of $y = 3x^3 + 5x^2 - 16x - 28$ by factoring

4. Sketch a polynomial function with
   - degree 4
   - zeros of -2 and 3, zero of 5 with multiplicity of 2
   - a negative leading coefficient.



5. True or false

a) The degree of a polynomial indicates the number of roots the equation has.

b) The range of all polynomial functions is the Reals.

c) A polynomial equation with an odd degree has at least one root.

d) An even degree polynomial with a positive leading coefficient has an overall maximum.