Warmup 3.3b

1. Factor the following:

   a) x^2-5x-6

   $(x-6)(x+1)$

   b) $3x^2-5x-2$

   -6

   $(x-2)(3x+1)$

   c) x^3-9x

   $x(x^2-9)$

   $x(x+3)(x-3)$

   d) x^4-81

   $(x^2+9)(x^2-9)$

   $(x^2+9)(x+3)(x-3)$

2. Which of the following are factors of $p(x)=x^3-x^2-14x+24$

   a) $x-1$ b) $x+1$ c) $x-2$

   d) $x+2$ e) $x-3$ f) $x+3$

x	$p(x)$
-4	0
-3	30
2	0
3	0
4	16

3. How could you use the results of #2 to help you factor $p(x)=x^3-x^2-14x+24$?

   $(x-3)(x-2)(x+4) = x^3-x^2-14x+24$

3.3b The Integral Zero Theorem

We now have an easy way of determining whether a binomial is a factor of a given polynomial. The question now is: "Which binomials should we test" or "Which values should we test to see if $\underbrace{p(a)=0}_{\text{remainder.}}$"?

x-a is a factor

It is helpful to notice that if $p(x)=x^3-x^2-14x+24$ and $p(a)=0$, then

$$a^3-a^2-14a+24=0$$
$$a^3-a^2-14a=-24$$
$$a\left(a^2-a-14\right)=-24$$

This means that a must be a factor of 24, and thus means that the only values for a that must be tested are $\pm1,\ \pm2,\ \pm3,\ \pm4,\ \pm6,\ \pm8,\ \pm12,\ \pm24$

> ***Integral Zero Theorem***
>
> If $x-a$ is a factor of a polynomial with integral coefficients, then a must be a factor of the constant term of the polynomial.

The Factor Property allows us to significantly narrow our search for **possible** factors. It means that the only factors involving integers we need to consider are the ones using factors of the constant term.

Example 1: Factor completely: $x^3+8x^2+11x-20$

The factors of 20: $\pm1,\ \pm2,\ \pm4,\ \pm5,\ \pm10,\ \pm20$

$p(-1)=-24$ $\quad\therefore\ x+1$ is not a factor of $x^3+8x^2+11x-20$

$p(1)=0$ $\quad\therefore\ x-1$ is a factor of $x^3+8x^2+11x-20$

At this point, you could now test other factors or you could use synthetic division to help find the other factors.

x-1

$$\begin{array}{r|rrrr} +1 & 1 & 8 & 11 & -20 \\ & \downarrow & 1 & 9 & 20 \\ \hline & 1 & 9 & 20 & 0 \end{array}$$

$(x-1)(x^2+9x+20)$

$(x-1)(x+4)(x+5)$

Example 2: Factor completely: $3x^3 - 10x^2 - 11x + 42$

$x+2$

$$\begin{array}{r|rrrr} -2 & 3 & -10 & -11 & 42 \\ & \downarrow & -6 & 32 & -42 \\ \hline & 3 & -16 & 21 & 0 \end{array}$$

x	$p(x)$
-2	0
3	0

$(x+2)$

$(x-3)$

63

$(x+2)(3x^2 - 16x + 21)$

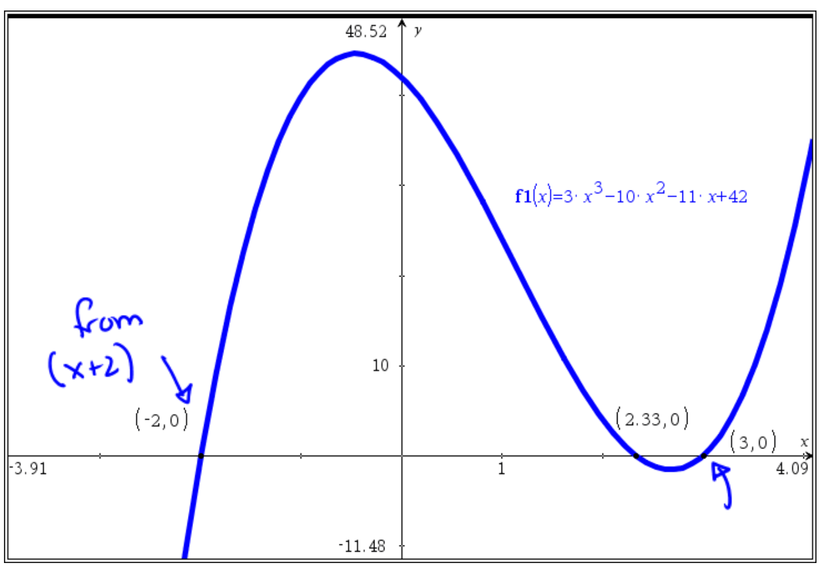
$(x+2)(\underbrace{3x^2 - 9x} \underbrace{-7x + 21})$

$(x+2)\left[3x(x-3) - 7(x-3)\right]$

$(x+2)(x-3)(3x-7)$ ✓

$(x+2)(x-3)(3x-7)$ ✓

Graph the function $f(x) = 3x^3 - 10x^2 - 11x + 42$. How could you use the factors of the polynomial to help determine the zeros of the function?



$(x+2)(x-3)(3x-7)$

$x=-2$

$x=3$

$3x-7=0$

$3x=7$

$x=2.\overline{3}$

Example 3: Factor completely: $x^4-2x^3-3x^2+8x-4$

The factors to test are: $\pm 1, \pm 2, \pm 4$

Since $p(1)=0$, we can say that $(x-1)$ is a factor.

Using synthetic division, we can obtain another factor.

$x-1$

$$\begin{array}{r|rrrrr} +1 & 1 & -2 & -3 & 8 & -4 \\ & \downarrow & 1 & -1 & -4 & 4 \\ \hline & 1 & -1 & -4 & 4 & 0 \end{array}$$

$(x-1)(x^3-x^2-4x+4)$

The other factor is x^3-x^2-4x+4. This can now be further factored using the Factor Theorem again or by trying to use Factoring by Grouping.

Factor Theorem	Factoring by Grouping
$x-2$ $\begin{array}{r\|rrrr} 2 & 1 & -1 & -4 & 4 \\ & & 2 & 2 & -4 \\ \hline & 1 & 1 & -2 & 0 \end{array}$ $(x-1)(x-2)(\underbrace{x^2+x-2})$ $(x-1)(x-2)(x+2)(x-1)$	$(x-1)(\underbrace{x^3-x^2}\underbrace{-4x+4})$ $(x-1)\left(x^2(x-1)-4(x-1)\right)$ $(x-1)(x^2-4)(x-1)$ $(x-1)(x+2)(x-2)(x-1)$

Optional: If you were going to extend your search for factors to include rationals, you could use the following:

Rational Zero Theorem: If $ax+b$, $cx+d$ and $ex+f$ were factors of the cubic polynomial $p(x)=mx^3+nx^2+px+q$, then $p(x)=(ax+b)(cx+d)(ex+f)$. This means that a, c, e must be factors of m and b, d, f must be factors of q. This means that in testing for factors, we might also incorporate the factors of the leading term and look also at substituting $\dfrac{\pm \textit{factors of constant}}{\textit{factors of leading coeff}}$

p133 #5, 8, 11, 13, C2, C3 * 15, 16.