3.1 Characteristics of Polynomial Functions

***Investigating the graphs of polynomial functions*.**

For each of the following functions, state the

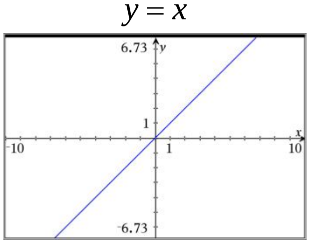
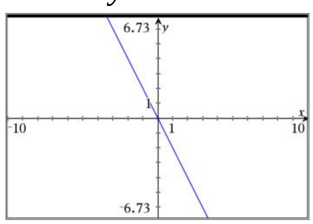
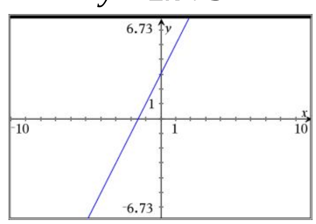
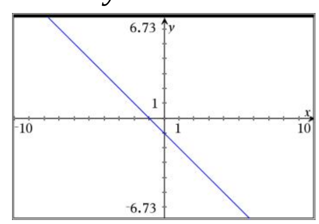
- i) degree of the function (the greatest power in the function)
- ii) end behaviour (the behaviour of the function as $|x|$ becomes very large)
- iii) y – intercept and the constant term
- iv) sign of the leading coefficient ←
- v) number of x – intercepts
- vi) number of maximums and minimums (peaks/valleys)
- vii) domain and range (this may requiring graphing the function on your calculator)

*leading coefficient is on the term with the highest power

Note also any similarities and differences between the graphs within each group.

Q: 2 | 1 / 3 | 4

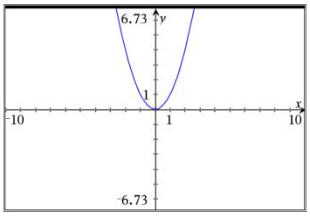
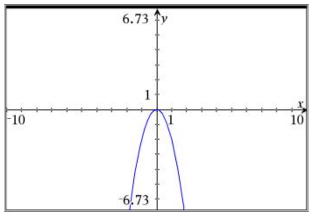
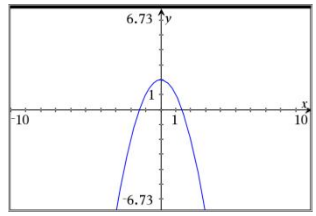
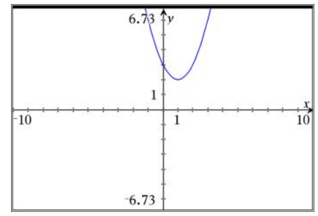
Linear Functions **Degree** 1

	$y = x$	$y = -2x$	$y = 2x + 3$	$y = -x - 1$
				
End behaviour	terminates in Q3 and Q1	Q2 and Q4	Q3 and Q1	Q2 and Q4
y – intercept Constant	0	0	3 3	-1 -1
Sign of coeff	positive	−	+	−
# x - int	1	1	1	1
# max/min	0	0	0	0
Domain Range	$\mathbb{R}$ $\mathbb{R}$	$\mathbb{R}$ $\mathbb{R}$	$\mathbb{R}$ $\mathbb{R}$	$\mathbb{R}$ $\mathbb{R}$

Similarities/differences – all linear functions have

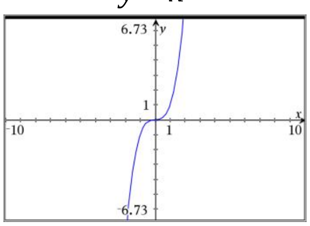
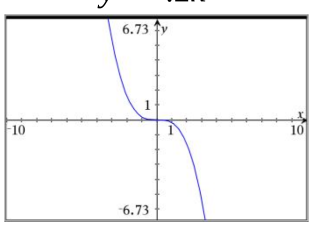
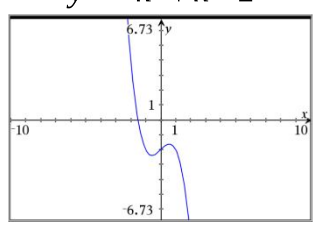
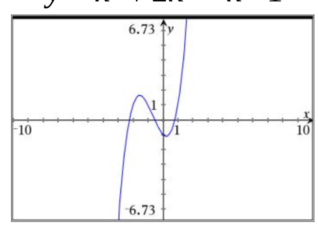
- no max or min
- domain = $\mathbb{R}$
- range = $\mathbb{R}$
- have one x-int
- terminate in quadrants 1,3 or 2,4 based on the sign of the leading coefficient

Quadratic Functions **_Degree_** 2

	$y = x^2$	$y = -2x^2$	$y = -x^2 + 2$	$y = x^2 - 2x + 3$
				
End behaviour	Q1 and Q2	Q3 / Q4	Q3 / Q4	Q1 / Q2
y – intercept Constant	0 0	0 0	2 2	3 3
Sign of coeff	+	–	–	+
# x - int	1	1	2	0
# max/min	1	1	1	1
Domain Range	ℝ $y \geq 0$	ℝ $y \leq 0$	ℝ $y \leq 2$	ℝ $y \geq 2$

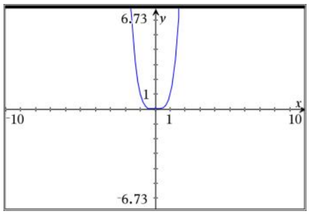
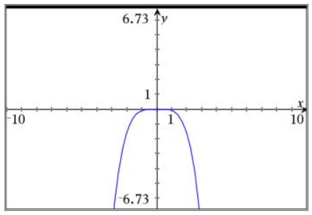
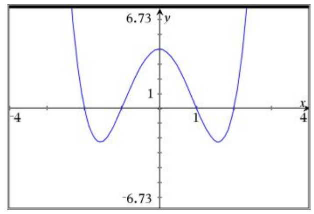
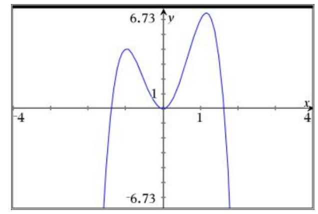
Similarities/differences exactly 1 max/min ; terminate Q1/Q2 or Q3/Q4
-yint is the constant term depending on sign
domain : x=ℝ

Cubic Functions **_Degree_** 3

	$y = x^3$	$y = -.2x^3$	$y = -x^3 + x - 2$	$y = x^3 + 2x^2 - x - 1$
				
End behaviour	Q3 / Q1	Q2 / Q4	Q2 / Q4	Q3 / Q1
y – intercept Constant	0 0	0 0	-2 -2	-1 -1
Sign of coeff	+	–	–	+
# x - int	1	1	1	3
# max/min	0	0	2	2
Domain Range	ℝ ℝ	ℝ ℝ	ℝ ℝ	ℝ ℝ

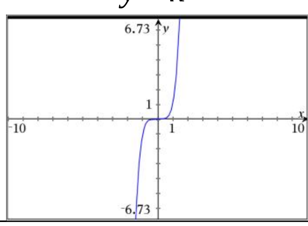
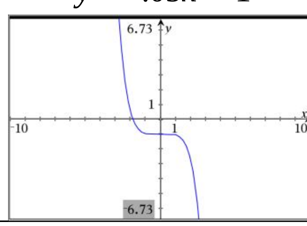
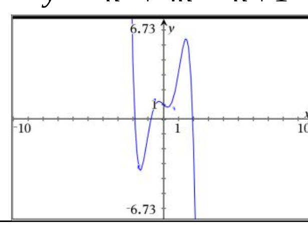
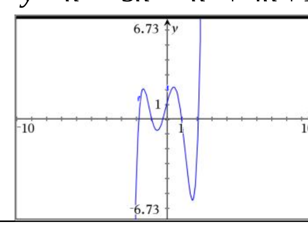
Similarities/differences

Quartic Functions ***Degree*** 4

	$y = x^4$	$y = -.1x^4$	$y = x^4 - 5x^2 + 4$	$y = -4x^4 + x^3 + 9x^2$
				
End behaviour	Q1 / Q2	Q3 / Q4	Q1 / Q2	Q3 / Q4
y – intercept Constant	0 0	0 0	4 4	0 0
Sign of coeff	+	−	+	−
# x - int	1	1	4	3
# max/min	1	1	3	3
Domain Range	ℝ $y \geq 0$	ℝ $y \leq 0$	ℝ $y \geq -2$	ℝ $y \leq 6.25$

Similarities/differences

Quintic Functions ***Degree*** 5

	$y = x^5$	$y = -.05x^5 - 1$	$y = -x^5 + 4x^3 - x + 1$	$y = x^5 - 5x^3 - x^2 + 4x + 1$
				
End behaviour	Q3 / Q1	Q2 / Q4	Q2 / Q4	Q3 / Q1
y – intercept Constant	0 0	-1 -1	1 1	1 1
Sign of coeff	+	−	−	+
# x - int	1	1	3	5
# max/min	0	0	4	4
Domain Range	ℝ ℝ	ℝ ℝ	ℝ ℝ	ℝ ℝ

Similarities/differences

What similarities are there in the graphs of polynomials with an odd degree?

- end behaviour based on leading coefficient. Q1/Q3 or Q2/Q4

domain: $\mathbb{R}$

range = $\mathbb{R}$

What similarities are there in the graphs of polynomials with an even degree?

end behaviour based on leading coefficient Q1/Q2 or Q3/Q4

domain: $\mathbb{R}$

range: varies, but is never $\mathbb{R}$

Definition: A polynomial function of degree n is any function of the form

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_1 x + a_0$$

where

- n is a whole number → degree
- x is the variable → have whole number exponents
- $a_n, a_{n-1}, a_{n-2}, \ldots a_1, a_0$ are real numbers. a_n is called the leading coefficient.
  a_0 is constant term

1. Identify which of the following are polynomial functions. Justify your answers.

Function	Polynomial?	Justification
$f(x) = 2\lvert x\rvert - 3$	no	graph is pointy ; can't have abs. value
$f(x) = 5^x + x^2$	no	5^x has a variable exponent
$f(x) = 3x^4 - \sqrt{2}\,x^3 + .7x^2 - x + 1059$	yes	coefficients are all real
$f(x) = \sqrt{x} + 3$	no	$\sqrt{x} = x^{1/2}$
$f(x) = -\frac{4}{7}x^5 + x^4 - \pi x^3 + \sqrt{8}$	yes	coefficients can be irrational.

Characteristics of Polynomial Functions

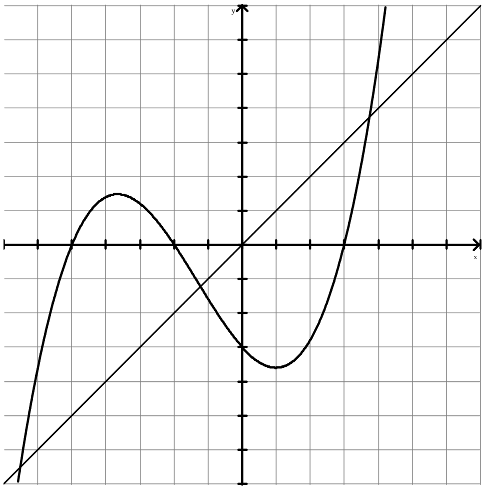
The graph of any polynomial function is a smooth, continuous curve. The domain of all polynomial functions is __all reals__. The y-intercept is the __constant__ __term__. The maximum number of x-intercepts is determined by the __degree__ of the polynomial. The greatest number of maxima and minima is one less than the degree. The end behaviour of the graph is determined by the whether the degree is __odd__ or __even__ , and also by the __sign__ of the leading coefficient. The orientation of the graph is determined by the __sign__ of the __leading__ coefficient.

$f(x) = x^4$ ← maximum of 4 solutions

$f(x) = x^3$ ← 3 x-int ⇒ 3 solutions

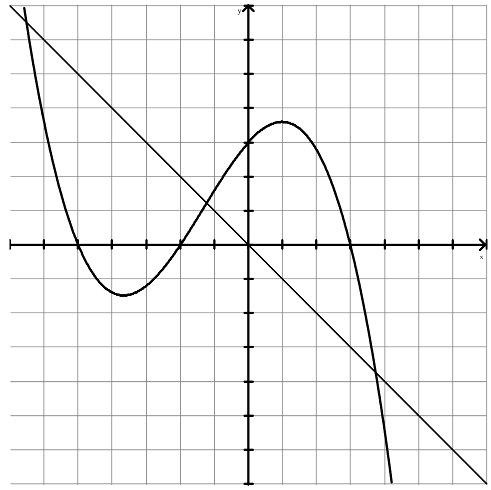
Odd Degree

Positive leading coefficient



Behaves like $y = x$

Negative leading coefficient

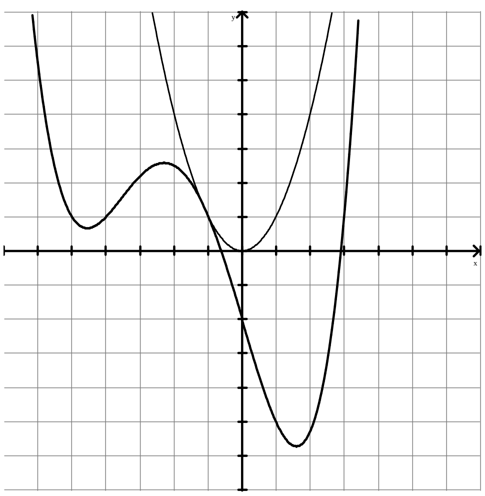


Behaves like $y = -x$

Must have at least one *x*-intercept, with a maximum of n-intercepts. The function does not have an overall maximum or minimum. Domain and the range is the $\{\text{Reals}\}$.

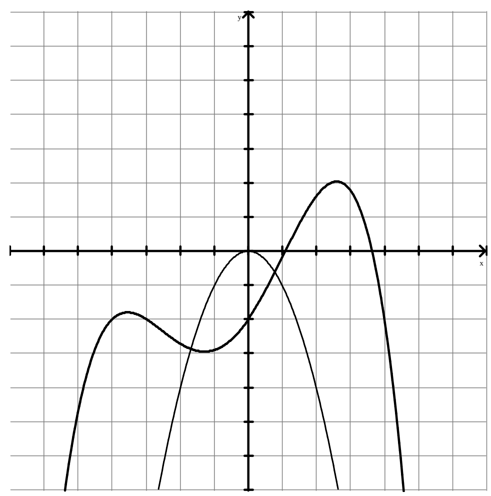
Even Degree

Positive leading coefficient



Behaves like $y = x^2$

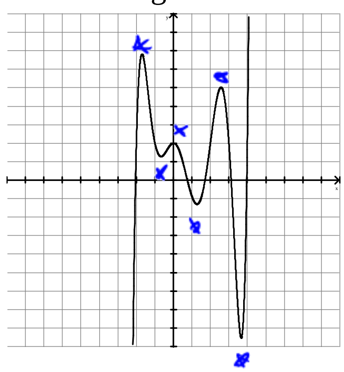
Negative leading coefficient



Behaves like $y = -x^2$

May have between zero and n *x*-intercepts. The function must have either an overall maximum or minimum. Domain is the $\{\text{Reals}\}$ and the range is determined by the maximum or minimum value.

2. What might you expect the degree of the following function to be? What would you expect the sign of the leading coefficient to be?



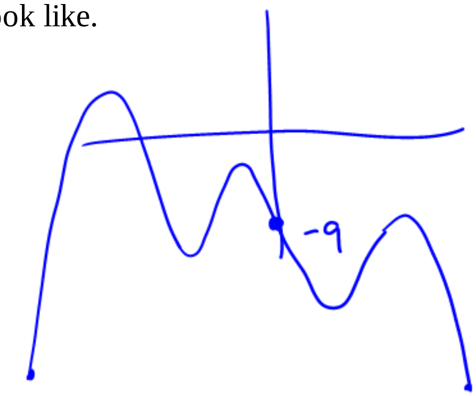
odd

6 max/min

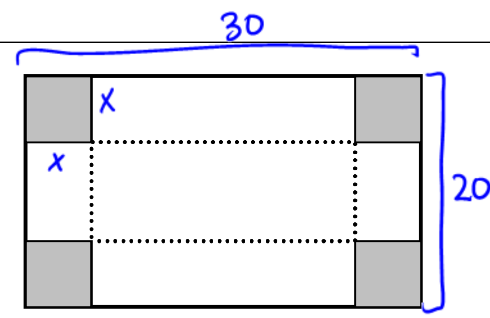
degree could be 7 or higher

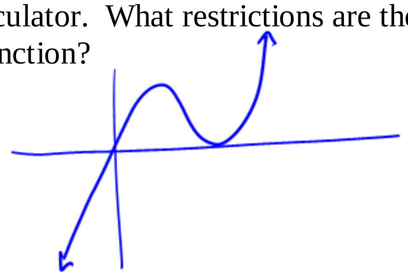
positive leading coefficient

3. Predict what you would expect the graph of $y = -2x^6 + 3x^5 - 7x^3 + x - 9$ to probably look like.



4. An open box is to be made from a piece of paper by cutting equal ~~corners~~ squares from each of the corners. The dimensions of the paper are 20 cm by 30 cm. If x represents the length of the square, in centimetres, then the volume can be represented with the function $V(x) = \underline{4x^3} - 100x^3 + 600x$. Make a sketch of what this function will look like, and then graph using your calculator. What restrictions are there on the domain of this function?



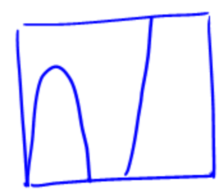


Xmin=0

Xmax=30

Ymin=0

Ymax=1500



domain: $0 \le x \le 10$

because I can only fit 2 10 cm squares along a 20 cm piece of paper.