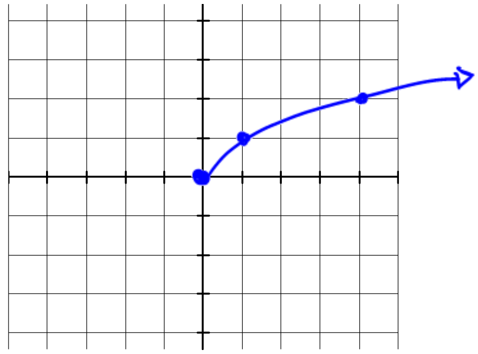
Radical Functions and Transformations

Draw the graph of $y = \sqrt{x}$:



Domain: $x \geq 0$

Range: $y \geq 0$

Recall what each value in the equation $y = -3(x+2)^2 - 5$ does to the graph of $y = x^2$

vertical stretched by factor of 3 and reflected then down 5

left 2 units horizontal.

Radical functions follow the form $y = a\sqrt{b(x-h)} + k$. Each value performs the following transformations on the standard graph of $y = \sqrt{x}$:

a: vertical stretch by a factor of a

b: horizontal stretch by a factor of $\frac{1}{b}$

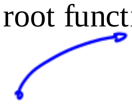
} includes a reflection if $a<0$ or $b<0$

h: h. translation "h" units right. eg $h=2 \rightarrow x-2$

k: v. translation "k" units up.

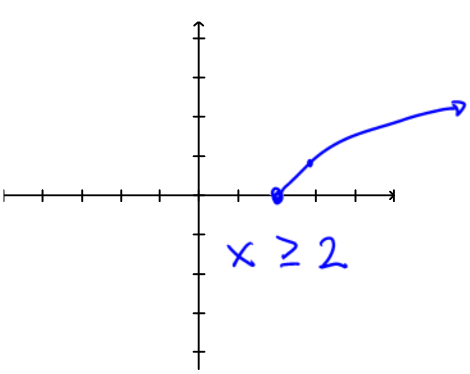
Using your knowledge of $y = \sqrt{x}$, sketch a graph of the following square root functions. State the domain of the function.

$\sqrt{x}$



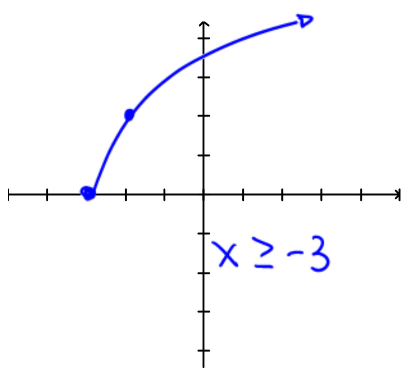
up 3

$y = \sqrt{x-2}$



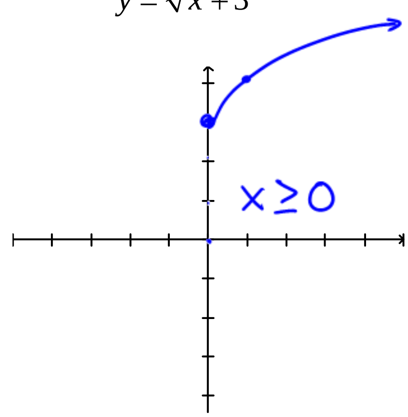
$x \geq 2$

$y = 2\sqrt{x+3}$



$x \geq -3$

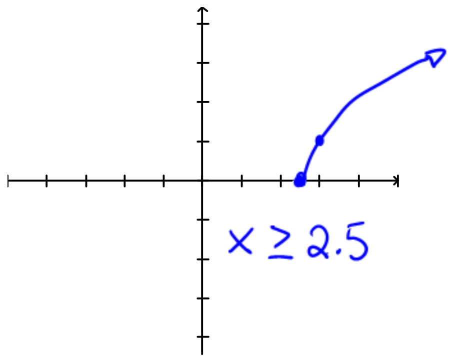
$y = \sqrt{x} + 3$



$x \geq 0$

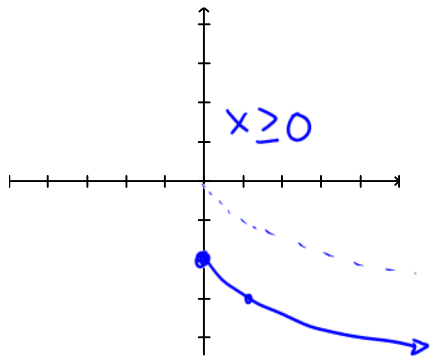
translation 5 right then h. stretch $\frac{1}{2}$

$y = \sqrt{2x - 5}$



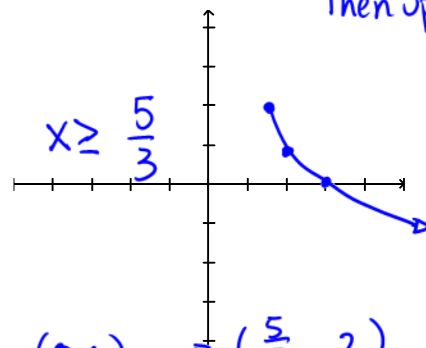
v. reflection in x-axis down 2

$y = -\sqrt{x} - 2$



right 5 then stretch by $\frac{1}{3}$

$y = -\sqrt{3x - 5} + 2$ reflection then up 2



$(0,0) \rightarrow (\frac{5}{3}, 2)$
$(1,1) \rightarrow (2, 1)$
$(4,2) \rightarrow (3, 0)$

Examples:

1. Using the given key points from the graph $y = \sqrt{x}$, map the new points for each given transformation.

Transformation of $y = \sqrt{x}$	**Mapping**
Vertical stretch by a factor of 4	(0,0) → (0,0) (1,1) → (1,4) (4,2) → (4,8) (9,3) → (9,12)
Horizontal reflection in the y-axis	(0,0) → (0,0) (1,1) → (-1,1) (4,2) → (-4,2) (9,3) → (-9,3)
Horizontal translation of 1 unit to the left and a vertical translation of 3 units up	(0,0) → (-1,3) (1,1) → (0,4) (4,2) → (3,5) (9,3) → (8,6)

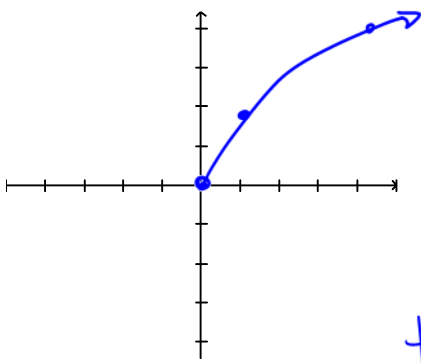
2. Apply mapping notation to the point (x, y) to determine a general mapping notation for the transformed function.

Transformation of $y = \sqrt{x}$	**Mapping**
Horizontal stretch by a factor of $\frac{1}{3}$	$(x, y) \rightarrow \left(\frac{x}{3}, y\right)$
Reflection in the y-axis	$(x, y) \rightarrow (-x, y)$
Vertical translation of 5 units down	$(x, y) \rightarrow (x, y-5)$

3. Graph each of the equations below. Describe what is special about the graphs.

v. stretch by factor of 2

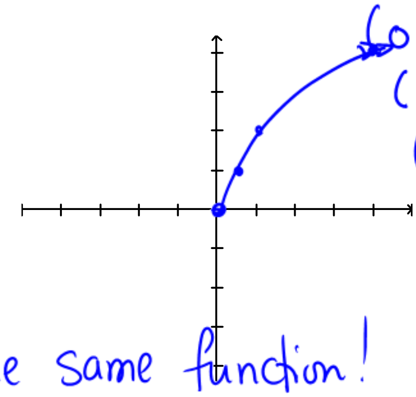
$y = 2\sqrt{x}$



$(0,0) \rightarrow (0,0)$
$(1,1) \rightarrow (1,2)$
$(4,2) \rightarrow (4,4)$

h. stretch by $\frac{1}{4}$

$y = \sqrt{4x}$



$(0,0) \rightarrow (0,0)$
$(1,1) \rightarrow (\frac{1}{4},1)$
$(4,2) \rightarrow (1,2)$
$(16,4) \rightarrow (4,4)$

these are the same function!

$$\sqrt{4x} = \sqrt{4} \cdot \sqrt{x}$$
$$= 2\sqrt{x}$$

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