Review Warmup

1. Using Newton's Method, determine a third approximation to a root of $x^3 - 3x^2 + 4.1 = 0$ if the initial approximation is $x = 2.1$.

$x_1 = 2.1$

$f(x) = x^3 - 3x^2 + 4.1$

$f'(x) = 3x^2 - 6x$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1.8920635$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 2.110278$$

2. For the function $y = x^3$, find the points on [0 , 3] which satisfy the Mean value Theorem.

① find overall slope

$$m_{sec} = \frac{f(3) - f(0)}{3-0} = \frac{27-0}{3-0} = 9$$

② set $m_{sec} = y'$

$9 = 3x^2$

$x^2 = 3$

$x = \pm\sqrt{3}$

③ $x = \sqrt{3}$

$y = (\sqrt{3})^3$

$(\sqrt{3}, \sqrt{3}^3)$

3. Determine the linear approximation to $y = \sqrt[3]{x}$ for x close to 8. Use the linear approximation to approximate $\sqrt[3]{7.9}$ to three decimal places.

$a =$ easy to figure out number

$(8,2)$ is $(a, f(a))$

$y = \sqrt[3]{x}$

$y' = \frac{1}{3}(x)^{-2/3}$

$f(x) = f'(a) \cdot (x-a) + f(a)$

$= \frac{1}{3}(8)^{-2/3} \cdot (7.9-8) + 2$

$= \frac{1}{12}(-.1) + 2$

$= 1.992$

4. Use differentials to calculate the approximate change in M when r changes from 20 to 19.9 and where $M = \frac{5}{r}$. Determine the actual change in M as well.

$M = \frac{5}{r} = 5 \cdot r^{-1}$

$\frac{dM}{dr} = \frac{-5}{r^2}$

approx

$dM = \frac{-5}{r^2} \cdot dr$

$= \frac{-5}{(20)^2}(19.9 - 20)$

$= 0.00125$

actual change

$\Delta M = M(19.9) - M(20)$

$= .00126$

5. Find dy if $y = \sin x + x^3$

$\frac{dy}{dx} = \cos x + 3x^2$

$dy = (\cos x + 3x^2) \cdot dx$