Review Warmup II

1. A spherical lollipop is licked so that its volume decreases at a rate of $12\text{ mm}^3/\text{min}$. How fast is the radius decreasing when the diameter is 30 mm?

$r = 15\text{mm}$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dr} = \frac{4\pi r^2}{1}$

want $\frac{dr}{dt} = \frac{dr}{dV} \cdot \frac{dV}{dt}$

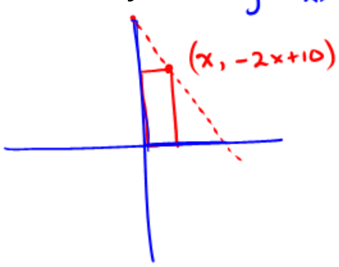
$= \frac{1}{4\pi r^2} \cdot -12 \Big|_{r=15}$

$= \frac{-12}{4\pi(15)^2}$

$= \frac{-12}{900\pi}$

$= -0.004\text{ mm/min}$

2. Find the dimensions of the rectangle with largest area bounded by the x and y axis and the line $2x + y = 10$ $\quad y = -2x+10$



$A = x(-2x+10)$

$A = -2x^2 + 10x$

$\frac{dA}{dx} = -4x + 10$

$0 = -4x + 10$

$4x = 10$

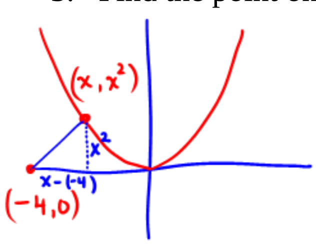
$x = \frac{10}{4} = 2.5$

$y = -2(2.5) + 10$

$y = 5$

$\boxed{2.5 \times 5}$

3. Find the point on the parabola $y = x^2$ that is closest to $(-4, 0)$.



$d^2 = (x^2)^2 + (x+4)^2$

$d^2 = x^4 + x^2 + 8x + 16$

$2d \cdot d' = 4x^3 + 2x + 8$

$d' = \frac{4x^3+2x+8}{2d}$

$d' = \frac{2x^3+x+4}{d} = 0$

$2x^3 + x + 4 = 0$

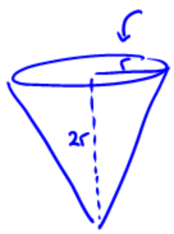
use graphing calc $\boxed{x = -1.128}$

$y = x^2$

$y = 1.272$

$(-1.128, 1.272)$

4. Water is poured at a rate of $20\text{ cm}^3/\text{s}$ into a conical container (vertex down) with a height that is twice its radius. How fast is the water rising when the depth of water in the container is 10 cm?



$$\frac{dV}{dt} = 20\ cm^3/s$$

$$h = 2r$$

$$\frac{dh}{dr} = 2$$

$$V = \frac{1}{3}\pi r^2 h$$

want $\left.\frac{dh}{dt}\right|_{h=10cm}$

$$V = \frac{1}{3}\pi r^2 (2r)$$

$$V = \frac{2\pi}{3} r^3$$

$$\frac{dV}{dr} = 2\pi r^2$$

$$\frac{dh}{dt} = \frac{dV}{dt}\,\frac{dr}{dV}\cdot\frac{dh}{dr}$$

$$= 20 \cdot \frac{1}{2\pi r^2} \cdot 2 \Big|_{r=5}$$

$$= \frac{40}{2\pi(5)^2}$$

$$= \frac{20}{25\pi} = \frac{4}{5\pi}$$

$$V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 \cdot h$$

$$V = \frac{1}{3}\pi\,\frac{h^2}{4}\,h$$

$$V = \frac{\pi}{12} h^3$$

$$\frac{dV}{dh} = \frac{3\pi h^2}{12}$$

$$\frac{dh}{dt} = \frac{dV}{dt}\cdot\frac{dh}{dV}$$

$$= 20 \cdot \frac{12}{3\pi(10)^2}$$

$$= \frac{240}{300\pi} = \frac{4}{5\pi}$$