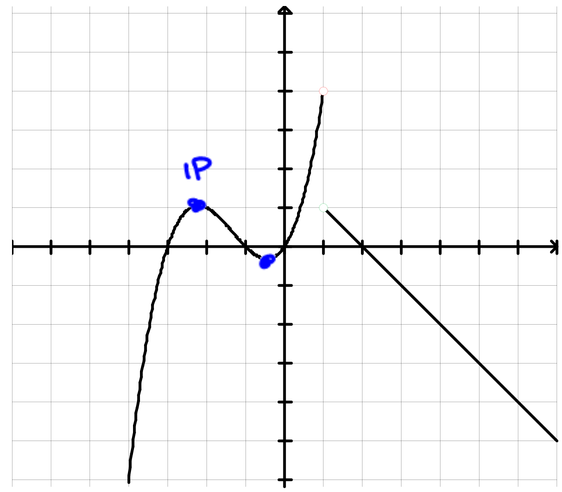
Unit 4.7 Warmup

1. Given the graph of $y = f'(x)$ to the right determine for the function $y = f(x)$ (Assume that the function $y = f(x)$ is a continuous function)



a) where any critical points occur, and whether they are maximum or minimum
b) intervals where $f(x)$ is increasing or decreasing
c) where any inflection points might occur
d) intervals where $f(x)$ is concave up; concave down

a) $x = -3$ local min ↘ ↗
$x = -1$ local max ↗ ↘
$x = 0$ local min ↘ ↗
$x = 1$
$x = 2$ local max ↗ ↘

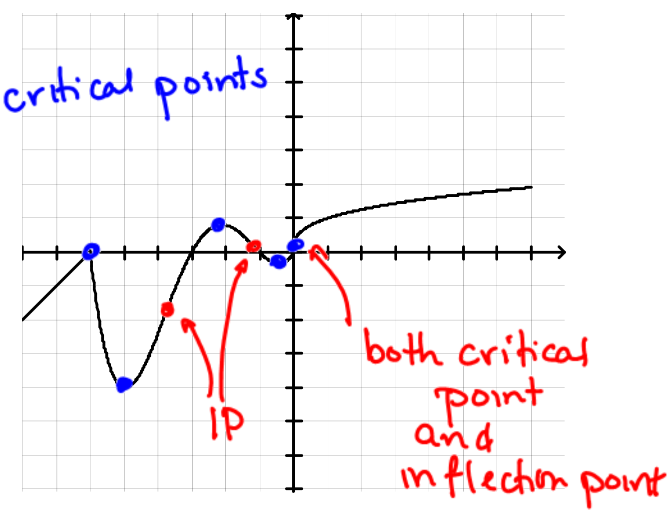
b) increasing $(-3,-1)$, $(0,1)$, $(1,2)$
decreasing $(-\infty,-3)$ $(-1,0)$ $(2,\infty)$

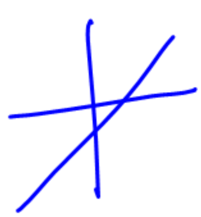
c) $x = -2.2$, $-.5$, 1

d) conc up $(-\infty,-2.2)$ $(-.5,1)$
conc down $(-2.1,-.5)$ $(1,\infty)$

2. How many critical points and inflection point does the graph below appear to have?

5 critical points





when $f'(x) = 0$ or undefined

3. Must all cubic function have *i*) a critical point? *ii*) an inflection point? Explain.

i) $y = ax^3 + bx^2 + cx + d$
$y' = 3a \cdot x^2 + 2bx + c$
it is possible that y' is not 0, ∴ this could have no critical point

ii) $y'' = 6ax + 2b$
this always has a solution for $y'' = 0$

Unit 4.7 Warmup II

1. Sketch a graph of a function with the following properties:

$f(x)$, $f'(x)$, $f''(x)$ are defined and continuous for all $x \neq 0$

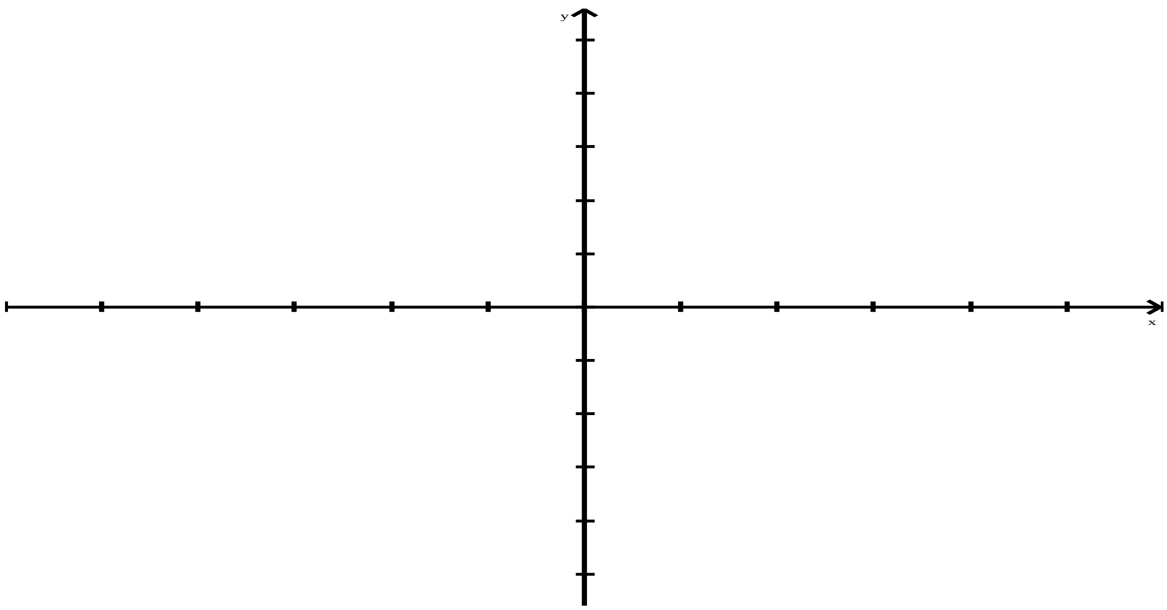
$\lim_{x\to\infty} f(x) = -\infty$ $\quad \lim_{x\to-\infty} f(x) = 2$, $\lim_{x\to 0} f(x) = \infty$

$f(-3) = f(-1) = f(2) = 0 \qquad f'(-2) = f'(-4) = f'(1) = 0$

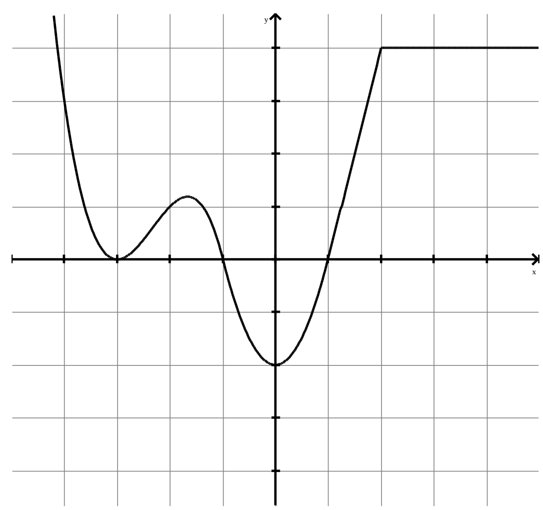
$-3, -1, 2$ are the only zeros of $f(x)$; $-4, -2, 1$ are the only zeros of $f'(x)$

$f''(x) < 0$ for all x in $(-5,-3)\cup(1,2)$; $f''(-5) = f''(-3) = f''(1) = f''(2) = 0$

$f''(x) > 0$ for all x in $(-\infty,-5)\cup(-3,0)\cup(0,1)\cup(2,\infty)$



2. Given the graph of $y = f'(x)$, determine intervals where $y = f(x)$ is increasing/decreasing and concave up/concave down. Also determine the *x* coordinates of any critical points or inflection points.



3. Find any extrema for the function $y = x^2 e^{\frac{1}{x}}$