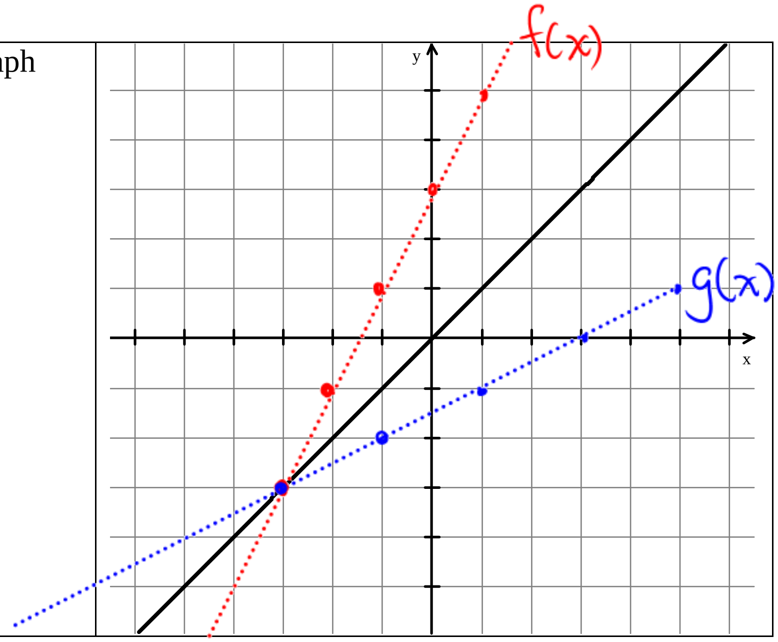
PC12 Warm-Up 1.4

Complete the table below and then graph the function $f(x) = 2x + 3$

x	y
−3	-3
−2	-1
−1	1
0	3
1	5
2	7
3	9



Now interchange the *x* and *y* coordinates for each ordered pair. Plot the points and draw the line passing through these points. We will call this function $g(x)$. Using the table of values, determine the equation of $g(x)$

x	y
-3	-3
-1	-2
1	-1
3	0
5	1
7	2
9	3

← invariant points

f(x) and g(x) are inverses

you transpose (switch) x and y values.

The line $y = x$ is also graphed on the grid above. What transformation would change the line $y = f(x)$ to the other line, $y = g(x)$

reflection in the line y = x

1.4 Inverse of a Relation

The inverse of a function is a rule for reversing what the function does, or it is a rule for changing the output of a function back to its input. We can find the inverse of a function or a relation by __switching__ x and y in the equation and then solving this equation for y.

Consider the function we've already graphed, $f(x) = 2x + 3$. What is the inverse of this function?

$y = 2x + 3$ original

$x = 2y + 3$

$x - 3 = 2y$

$y = \frac{x-3}{2}$ or $y = \frac{1}{2}x - \frac{3}{2}$

We can't call this function $f(x)$, since $f(x) = 2x + 3$ already, so we'll call this __$f^{-1}(x)$__ to show the inverse relationship between __$f(x)$__ and __$f^{-1}(x)$__

The notation $f^{-1}(x)$ is read as the __inverse__ of $f(x)$ or f inverse of x. Note that this is a different meaning for the superscript – in this situation it does not mean raising to a power of -1. Put another way, $f^{-1}(x) \neq \frac{1}{f(x)}$.

The inverse can also be denoted using the notation $x = f(y)$

In general, the inverse of a relation can be determined by doing any one of the following:

1. Interchanging the x and y coordinates for each ordered pair.
2. Interchanging x and y in the equation and then solving for y
3. Reflecting the graph of the relation in the line $y = x$

$(2,-1) \rightarrow (-1,2)$

Example 1. Given the graph of the relation to the right

a) What are the domain and range?

D: $[-4,4]$ R: $[-1,3]$

b) Graph the inverse on the same grid

c) What are the domain and range of the inverse?

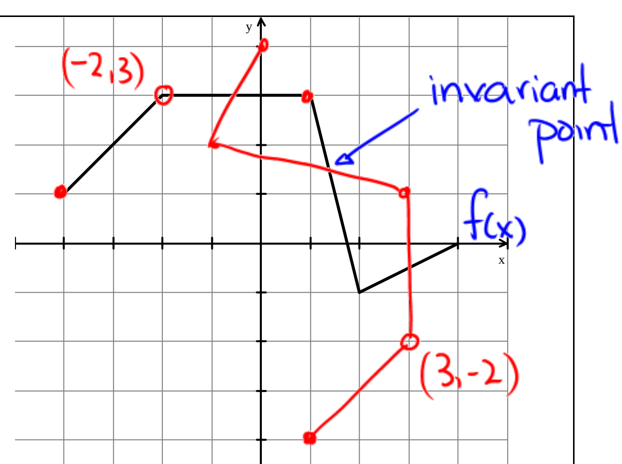
D: $[-1,3]$ R: $[-4,4]$

d) Is the inverse a function?

in this case, no

e) Are there any invariant points?

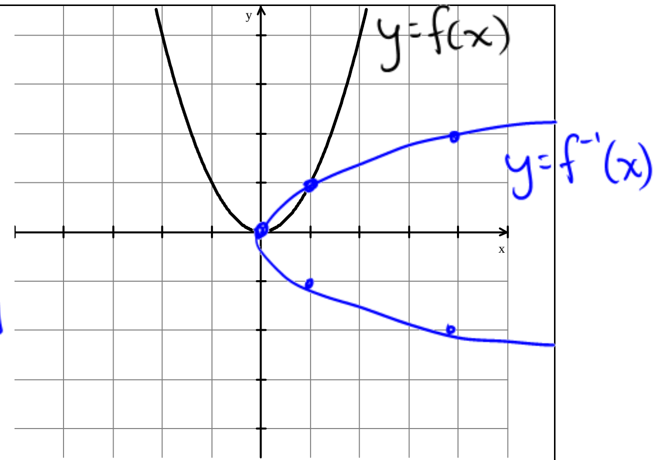
yes



original $-4 \leq x \leq 4$

inverse $-4 \leq y \leq 4$

Example 2. Graph the inverse of $y = x^2$ on the grid to the right.



a) What are the domain and range of the original function and the inverse?

original

D: $[-\infty,\infty]$ R: $[0,\infty]$

inverse

D: $[0,\infty]$

R $[-\infty,\infty]$

b) Is the inverse a function?

no

c) What is the equation of the inverse?

$y=x^2$

$x=y^2$

$y=\pm\sqrt{x}$

$y=\sqrt{x}$

$y=-\sqrt{x}$

Some observations:

1) The domain of a relation is the range of its inverse, and the range of the relation is the domain of its inverse.

2) The inverse of a function is not necessarily a function. In order for the inverse of a function to also be a function, the original function must pass the **horizontal line** test. This test simply says that if any horizontal line passes through a function in at least 2 points, then the inverse is not a function.

3) The invariant points lie on the line $y = x$

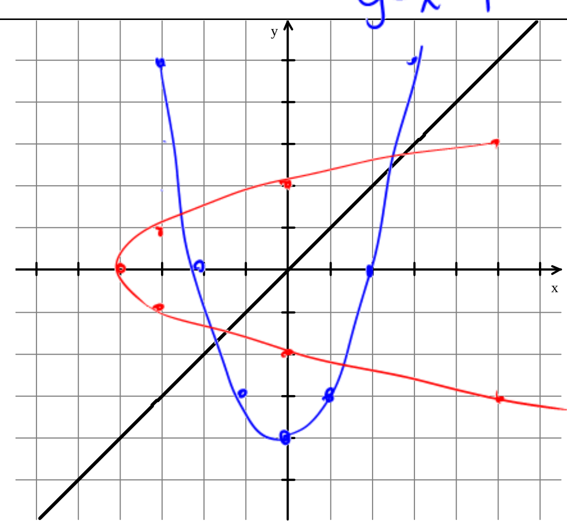
4) The inverse can always be denoted as $x = f(y)$. In the case when the inverse is also a function, then and only then can the inverse be denoted as $y = f^{-1}(x)$

5) To reflect a graph of a relation in the line $y = x$, interchange x and y in the equation.

Restricting the domain

$y=x^2-4$

Graph the function $y = x^2 - 4$ and its inverse on the grid below.



$y=f(x)$ fails Horizontal line test,

so $f^{-1}(x)$ is not a function.

Sometimes when the inverse of a function is not a function, the domain of the original function is **restricted** so that the inverse is also a function. In the example, what restriction could you place on the domain of the original so that above the inverse is also a function?

$f(x) = x^2 - 4 \; ; \; x \le 0$

Graph the function with the restricted domain, along with its inverse.

D: $[-\infty, 0]$

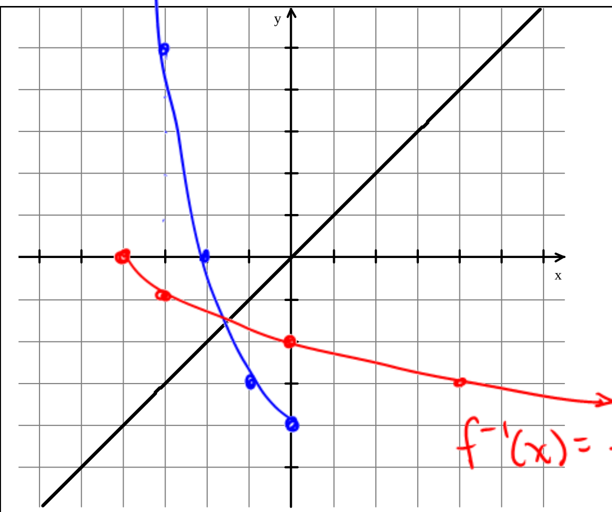
What would the equation of the inverse now be?

$y = x^2 - 4$

$x = y^2 - 4$

$x + 4 = y^2$

$y = \pm\sqrt{x+4}$



$f^{-1}(x) = -\sqrt{x+4}$

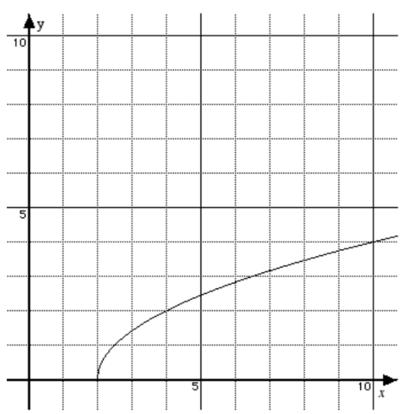
What other restriction could you have used in this question?

$x \ge 0$ would graph right half of parabola and the inverse $f^{-1}(x) = \sqrt{x+4}$

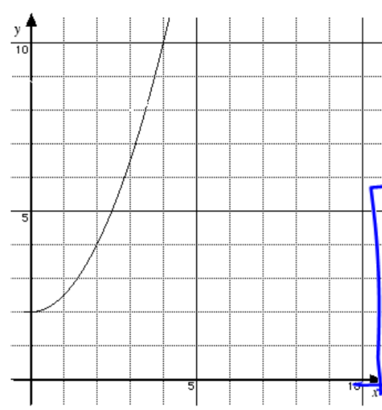
<u>Example 3:</u>

The graph on the right is the reflection (in the line $y = x$) of the graph on the left. Write the equation of the graph on the right.

$y = \sqrt{2x - 4}$



$x = \sqrt{2y - 4}$



$x^2 = 2y - 4$

$x^2 + 4 = 2y$

$\boxed{y = \frac{1}{2}x^2 + 2 \quad x \ge 0}$