2.13 Review Warmup

1. Determine $\frac{dy}{dx}$ for:

a) $y=\sin^3(5x+3)$

$y' = 3\sin^2(5x+3)[\cos(5x+3)]\cdot 5$

$= 15\sin^2(5x+3)\cos(5x+3)$

b) $y=\sqrt{\sin^2 x+x^3}$

$y' = \frac{1}{2}(\sin^2 x+x^3)^{-\frac{1}{2}}[2\sin x\cos x+3x^2]$

$y' = \dfrac{2\sin x\cos x+3x^2}{2\sqrt{\sin^2 x+x^3}}$

2. Determine $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for the relation: $xy=2y-3x^2$

$1\cdot y + xy' = 2y' - 6x \longrightarrow y' + 1y' + xy'' = 2y'' - 6$

$y+6x = 2y' - xy'$

$\boxed{y' = \dfrac{y+6x}{2-x}}$

$2y'+6 = 2y'' - xy''$

$\boxed{y'' = \dfrac{2y'+6}{2-x}}$

3. Find the equation of the tangent(s) to the relation in #2 at the points where $y=1$

1st derivative.

$y' = \dfrac{y+6x}{2-x}\Big|_{y=1}$

$x(1) = 2(1) - 3x^2$

$3x^2 - x - 2 = 0$

$(3x-2)(x+1) = 0$

$x = \frac{2}{3}$ or -1

$(-1, 1)$

$y' = \dfrac{1+6(-1)}{2-(-1)}$

$y' = \frac{-5}{3}$

$y-1 = \frac{-5}{3}(x+1)$

$(\frac{2}{3}, 1)$

$y' = \dfrac{1+6(\frac{2}{3})}{2-(2/3)}$

$y' = \dfrac{5}{\frac{4}{3}} = \frac{15}{4}$

$y-1 = \frac{15}{4}(x-\frac{2}{3})$

4. If a position function $s(t)$ had the property that $\boxed{v=\sqrt{s}}$, show that acceleration must be constant

$v = \frac{ds}{dt} = \sqrt{s}$

$a = \frac{dv}{dt}$

$a = \frac{dv}{dt}$

$\frac{dv}{ds} = \frac{1}{2}s^{-\frac{1}{2}}$

$a = \frac{dv}{dt} = \frac{dv}{ds}\cdot\frac{ds}{dt} = \frac{1}{2\sqrt{s}}\cdot\sqrt{s} = \frac{1}{2}$

5. If $y = m\sin p\cancel{t} + n\cos p\cancel{t}$ where *m*, *n*, and *p* are constants, then what is $\frac{d^2y}{dx^2}$?

$y' = m\cos px\,(p) + n(-\sin px)(p)$

$= mp\cos px - np\sin px$

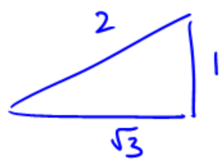
$y'' = mp(-\sin px)(p) - np(\cos px)p$

$= -mp^2\sin px - np^2\cos px$

Unit 2 Review Warmup

1. Determine $\lim_{h \to 0} \frac{\tan\left(\frac{\pi}{6}+h\right)-\tan\frac{\pi}{6}}{h}$

$$\left.\frac{d(\tan x)}{dx}\right|_{x=\frac{\pi}{6}} = \left.\sec^2(x)\right|_{x=\frac{\pi}{6}}$$



$$= \sec^2\frac{\pi}{6}$$

$$= \left[\frac{1}{\left(\frac{\sqrt{3}}{2}\right)}\right]^2 = \frac{4}{3}$$

2. A rock is thrown into the air form a cliff that is 10m high. For any time $t \geq 0$, its height is given by $h(t) = -5t^2 + 40t + 10$.

a) What is the maximum height reached? When $h' = 0$

$h'(t) = -10t + 40$

$0 = -10t + 40$

$\underline{\underline{t = 4}}$

$h(4) = -5(4)^2 + 40(4) + 10$

$h(4) = 90m$

b) What is the velocity when it hits the ground?

$h(t) = 0$ $\quad -5(t^2 - 8t - 2)$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{8 \pm \sqrt{64 - 4(1)(-2)}}{2}$

$= \frac{8 \pm \sqrt{72}}{2}$

$= 8.24$

$h'(8.24) = -10(8.24) + 40$

$= \underline{\underline{-42.4}}$

3. Determine $\frac{dy}{dx}$ if $y = \cos(xy)$

$y' = -\sin(xy) \cdot [1y + xy']$

$y' = -y\sin xy + xy' \cdot (-\sin(xy))$

$y' + xy'\sin(xy) = -y\sin xy$

$y' = \frac{-y\sin(xy)}{1 + x\sin(xy)}$

4. Find the tangent to the curve $y^2x + x + y = 17$ at the point where $y = 2$

$2yy'x + y^2 + 1 + y' = 0$

$y'(2xy+1) = -1 - y^2$

$y' = \left.\frac{-1-y^2}{2xy+1}\right|_{(x,y)=(3,2)} = \frac{-1-4}{12+1} = \frac{-5}{13}$

$y = 2$

$(2)^2x + x + 2 = 17$

$5x = 15$

$x = 3.$

$$\boxed{y - 2 = \frac{-5}{13}(x-3)}$$

Unit 2 Review

Determine the derivative of the following functions

1. $f(x) = \overset{f}{(4x+1)}\overset{g}{(1-x)^3}$

   $(4)(1-x)^3 + 3(1-x)^2(-1)\cdot(4x+1)$

   $(1-x)^2(4(1-x) - 3(4x+1))$

   $(1-x)^2(4-4x-12x-3)$

   $(1-x^2)(1-16x)$

2. $y = \dfrac{2-x}{3x+1}$

3. $y = \sqrt{3-2x}$

4. $y = \dfrac{2}{(5x+1)^3}$

5. $y = 3x^{2/3} - 4x^{1/2} - 2$

6. $y = 2\sqrt{x} - \dfrac{1}{2\sqrt{x}}$

7. $y = \sqrt{x^2+2x+1}$

8. $y = \dfrac{x}{\sqrt{1-x^2}}$

9. $y = \dfrac{1+x^2}{1-x^2}$

For questions 10-17, functions f and g have the values shown in the table

x	f	f'	g	g'
0	2	1	5	–4
1	3	2	3	–3
2	5	3	1	–2
3	10	4	0	–1

10. If $A = f + 2g$, then $A'(3) =$

11. If $B = f \bullet g$ then $B'(2) =$

12. If $D = \dfrac{1}{g}$, then $D'(1) =$

13. If $H(x) = \sqrt{f(x)}$, then $H'(3) =$

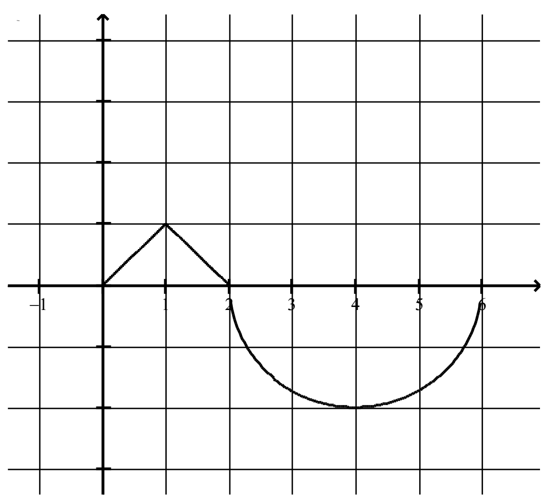
14. If $K(x) = \dfrac{f(x)}{g(x)}$, then $K'(0) =$

15. If $M(x) = f(g(x))$, then $M'(1) =$

16. If $P(x) = f(x^3)$, then $P'(1) =$

17. If $S(x) = f^3(f(x))$, then $S'(0) =$

Use the graph to answer questions 18-20
The graph consists of two line segments and a semicircle.



18. $f'(x) = 0$ for $x =$

19. $f'(x)$ does not exist for $x =$

20. $f'(5) =$

Determine $\frac{dy}{dx}$ for the following implicitly defined functions

21. $x^3 - xy + y^3 = 1$

22. $x + \cos(x+y) = 0$

23. $\sin x - \cos y - 2 = 0$

24. $3x^2 - 2xy + 5y^2 = 1$

25. If $f(x) = x^4 - 4x^3 + 4x^2 - 1$, for what values of x does graph have a horizontal tangent?

26. If $f(x) = 16\sqrt{x}$ then $f''(4) =$

27. If a point moves on the curve $x^2 + y^2 = 25$, then at $(0,5)$, $\frac{d^2y}{dx^2}$ is ____________

28. If $y = a\sin ct + b\cos ct$ where a, b, and c are constants, then $\frac{d^2y}{dt^2}$ is ____________

29. If $f(v) = \sin v$ and $v = g(x) = x^2 - 9$, then $(f \circ g)'(3) =$____________

30. If $f(x) = \frac{x}{(x-1)^2}$, then for what values of x does $f'(x)$ exist?

31. If $y = \sqrt{x^2+1}$, then what is the derivative of y^2 with respect to x^2?

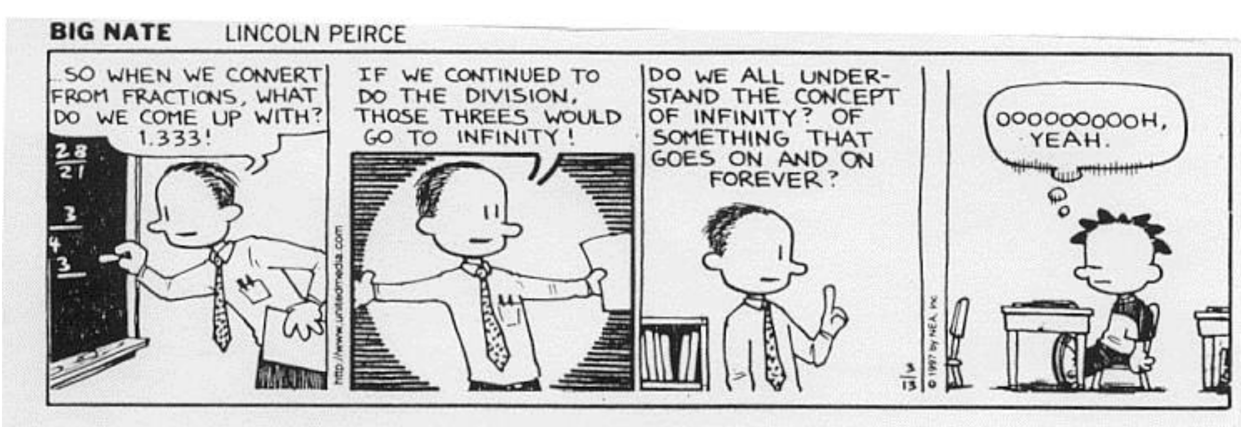
32. If $f(x) = \frac{1}{x^2+1}$ and $g(x) = \sqrt{x}$, then the derivative of $f(g(x))$ is ?

33. $\lim_{h \to 0} \frac{(1+h)^6 - 1}{h} =$ (Hint: Think about the definition of the derivative)

34. $\lim_{h \to 0} \frac{\sqrt[3]{8+h} - 2}{h} =$

35. $\lim_{x \to 0} \frac{\cos x - 1}{x} =$

36. If $f(a) = f(b) = 0$ and $f(x)$ is continuous on $[a,b]$, then
   a) $f(x)$ must be identically zero
   b) $f'(x)$ may be different from zero for all x on $[a,b]$
   c) there exists at least one number c, $a < c < b$, such that $f'(c) = 0$
   d) $f'(x)$ must exist for every x on (a,b)
   e) none of the preceding is true



37. Suppose $\lim_{x \to 0} \frac{g(x) - g(0)}{x} = 1$. It follows necessarily that

a) g is not defined at $x = 0$
b) g is not continuous at $x = 0$
c) The limit of $g(x)$ as x approaches 0 equals 1.
d) $g'(0) = 1$
e) $g'(1) = 0$

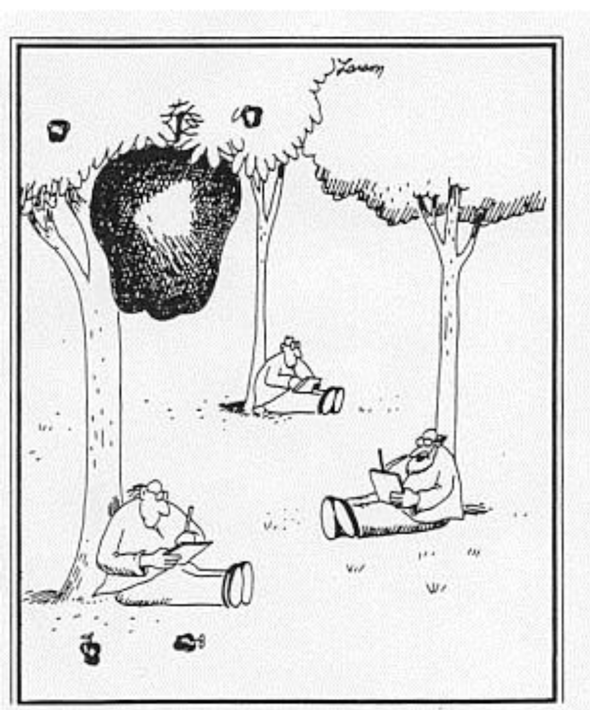
38. Find the instantaneous rate of change of the area of a circle with respect to a) its radius, r
b) its circumference, C

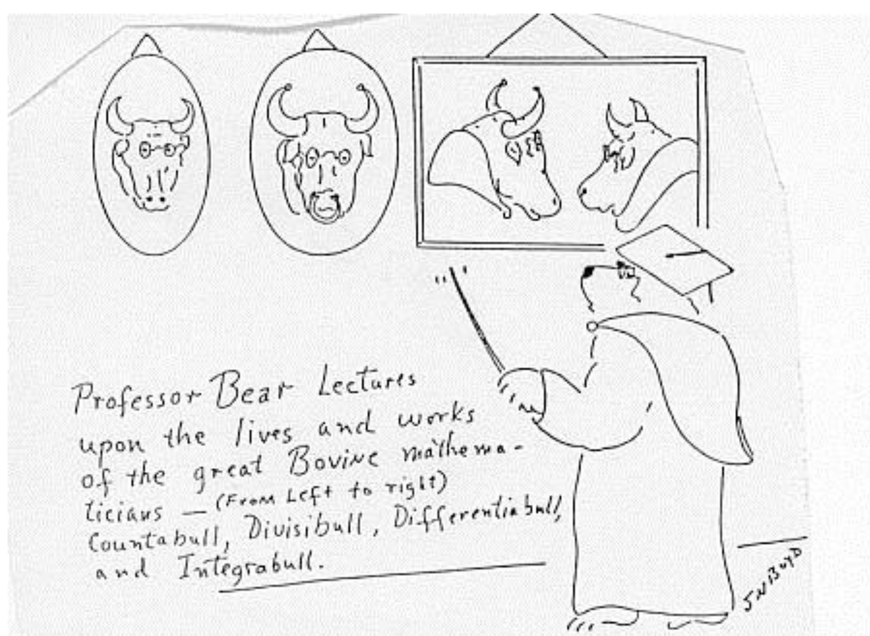
39. A particle moves along a straight line with position function $s(t) = 3t^3 - 11t^2 + 8t$. In what interval of t is the particle moving to the left along the line?

40. A ball is thrown directly upward with an initial velocity of 24.5 m/s and its height, h, in metres, is given by $h(t) = 24.5t - 4.9t^2$, where t is in seconds.

a) Find the velocity after 1 s.
b) When does it reach its maximum height?
c) What is its maximum height?
d) When does it strike the ground?
e) What is the acceleration?

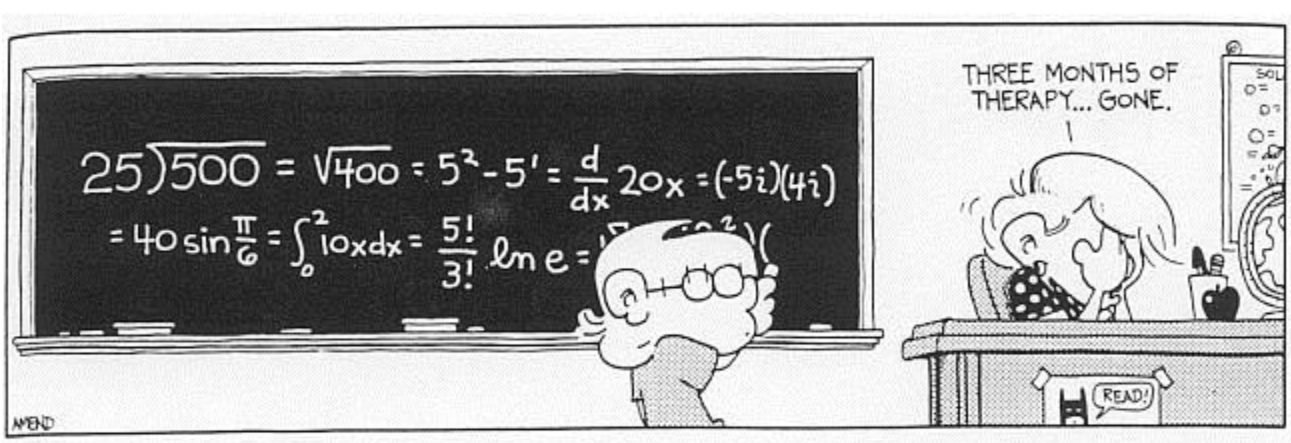
41. If the k^{th} derivative of $(3x - 2)^4$ is zero, then what must be the case concerning k ?


"Nothing yet. ... How about you, Newton?"



Key

1. $(1-x)^2(1-16x)$	2. $-\dfrac{7}{(3x+1)^2}$	3. $-\dfrac{1}{\sqrt{3-2x}}$	4. $-30(5x+1)^{-4}$
5. $2x^{-\frac{1}{3}}-2x^{-\frac{1}{2}}$	6. $\dfrac{1}{\sqrt{x}}+\dfrac{1}{4x\sqrt{x}}$	7. $\dfrac{x+1}{\sqrt{x^2+2x+1}}$	8. $\dfrac{1}{(1-x^2)^{\frac{3}{2}}}$
9. $\dfrac{4x}{(1-x^2)^2}$	10. 2	11. -7	12. $\dfrac{1}{3}$
13. $\dfrac{2}{\sqrt{10}}$	14. $\dfrac{13}{25}$	15. -12	16. 6
17. 225	18. 4 only	19. 1, 2, and 6	20. $\dfrac{1}{\sqrt{3}}$
21. $\dfrac{y-3x^2}{3y^2-x}$	22. $\csc(x+y)-1$	23. $-\csc y\cos x$	24. $\dfrac{y-3x}{5y-x}$
25. $\{0,1,2\}$	26. $-\dfrac{1}{2}$	27. $-\dfrac{1}{5}$	28. $-c^2y$
29. 6	30. reals except $x=1$	31. 1	32. $-(x+1)^{-2}$
33. 6	34. $\dfrac{1}{12}$	35. 0	36. B
37. D	38a) $2\pi r$	b) $\dfrac{C}{2\pi}$	For those of you looking for secret messages, here it is: Make sure you ask Mr. O what it is.
39. $\dfrac{4}{9}<t<2$	40a) 14.7 *m/s*	b) 2.5 *s*	c) 30.6 *m*
d) after 5 s	e) $-9.8\ m/s^2$	41. $k\geq 5$	



Derivatives

Definition of the Derivative: $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ $\qquad f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

Derivative of a Constant

$$\frac{d}{dx} k = 0$$

Power Rule

$$\frac{d}{dx} x^n = n\, x^{n-1}$$

Constant times a Function

$$\frac{d}{dx} k \bullet f(x) = k \bullet f'(x)$$

Product Rule

$$\frac{d}{dx} f \bullet g = f'\, g + f\, g'$$

Quotient Rule

$$\frac{d}{dx} \frac{f}{g} = \frac{f'\, g - g'\, f}{g^2}$$

Chain Rule: $\frac{d}{dx} f\left(g(x)\right) = f'\left(g(x)\right) \bullet g'(x)$ or $\frac{dy}{dx} = \frac{dy}{du} \bullet \frac{du}{dx}$

Derivatives of Trigonometric Functions:

$$\frac{d}{dx} \sin u = \cos u \bullet \frac{du}{dx} \qquad \frac{d}{dx} \csc u = -\csc u \cot u \bullet \frac{du}{dx}$$

$$\frac{d}{dx} \cos u = -\sin u \bullet \frac{du}{dx} \qquad \frac{d}{dx} \sec u = \sec u \tan u \bullet \frac{du}{dx}$$

$$\frac{d}{dx} \tan u = \sec^2 u \bullet \frac{du}{dx} \qquad \frac{d}{dx} \cot u = -\csc^2 u \bullet \frac{du}{dx}$$

Derivatives of Inverse Trigonometric Functions

$$\frac{d}{dx} \sin^{-1} u = \frac{d}{dx} \arcsin u = \frac{1}{\sqrt{1 - u^2}} \bullet \frac{du}{dx} \quad |u| < 1 \qquad \frac{d}{dx} \cos^{-1} u = \frac{d}{dx} \arccos u = \frac{-1}{\sqrt{1 - u^2}} \bullet \frac{du}{dx} \quad |u| < 1$$

$$\frac{d}{dx} \tan^{-1} u = \frac{d}{dx} \arctan u = \frac{1}{1 + u^2} \bullet \frac{du}{dx} \qquad \frac{d}{dx} \cot^{-1} u = \frac{d}{dx} \operatorname{arccot} u = \frac{-1}{1 + u^2} \bullet \frac{du}{dx}$$

$$\frac{d}{dx} \sec^{-1} u = \frac{d}{dx} \operatorname{arcsec} u = \frac{1}{|u| \bullet \sqrt{u^2 - 1}} \bullet \frac{du}{dx} \qquad \frac{d}{dx} \csc^{-1} u = \frac{d}{dx} \operatorname{arccsc} u = \frac{-1}{|u| \bullet \sqrt{u^2 - 1}} \bullet \frac{du}{dx}$$

Exponential Functions: $\frac{d}{dx} e^u = e^u \frac{du}{dx}$ $\qquad \frac{d}{dx} a^u = a^u \ln a \frac{du}{dx}$

Logarithmic Functions: $\frac{d}{dx} \ln u = \frac{1}{u} \frac{du}{dx}$ $\qquad \frac{d}{dx} \log_b u = \frac{1}{u \ln b} \frac{du}{dx}$