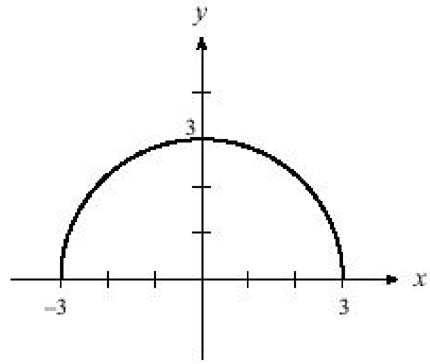
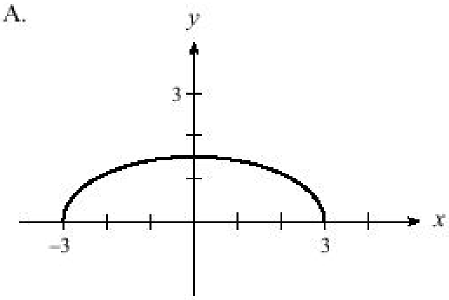
Math 12 Pre-Calculus
Warm-Up

1. The graph of $y = \sqrt{9 - x^2}$ is shown to the right.

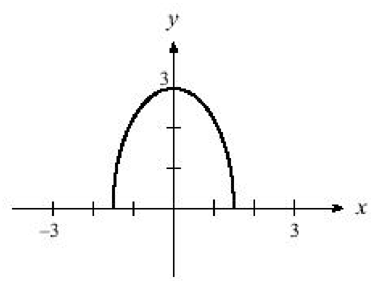


Write the equation for each of the following graphs that are transformations of $y = \sqrt{9 - x^2}$.

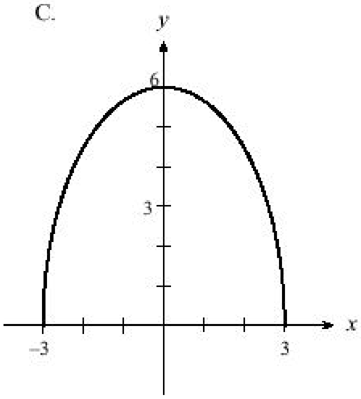
A.



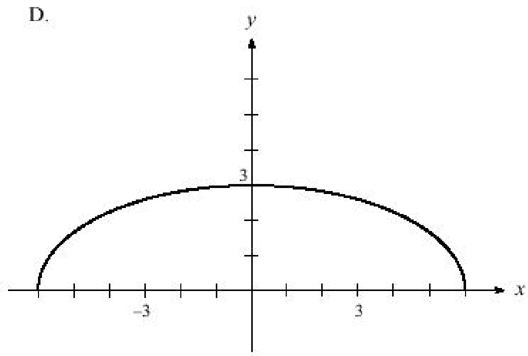
B.



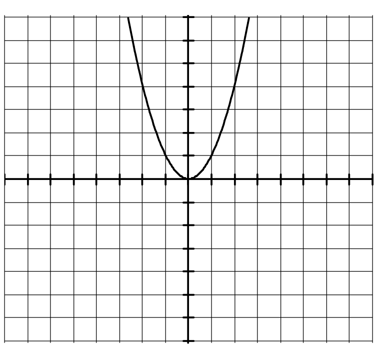
C.

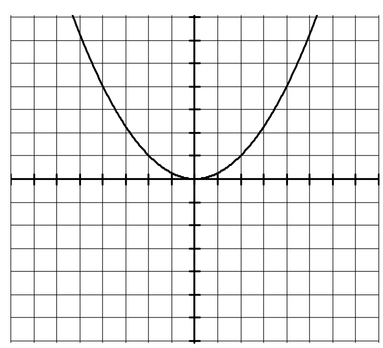


D.



2. What change might have occurred to produce the transformed graph to the right?





1.3 Combining Transformations

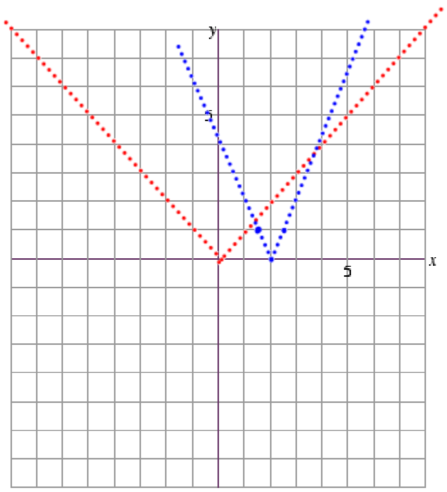
Part 1

Sketch the graph of $y = |x|$ on the grid below. Then, perform the following transformations on the function.

a) Translate 4 units to the right.

b) Compress horizontally by a factor of $1/2$.

Write the equation of the transformed function, and sketch its graph on the same grid below.



$(-1,1) \rightarrow (3,1) \rightarrow (1.5,1)$

$(0,0) \rightarrow (4,0) \rightarrow (2,0)$

$(1,1) \rightarrow (5,1) \rightarrow (2.5,1)$

Equation of the transformed function:

$y=f(x) \rightarrow f(x-4) \rightarrow f(2x-4)$

$y=|x| \rightarrow y=|x-4| \rightarrow y=|2x-4|$

$x \rightarrow x-4$ $\qquad$ $x \rightarrow 2x$

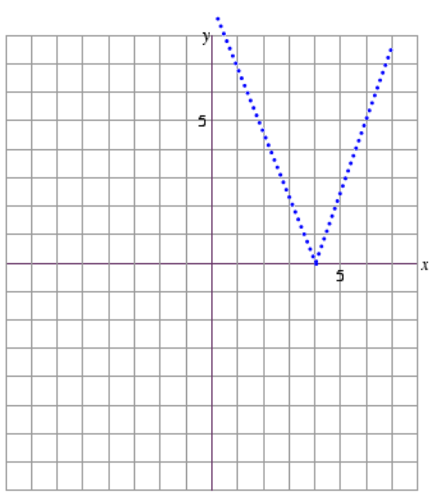
Part 2

As in Part 1, sketch the graph of $y = |x|$ on the grid below. This time, however, perform the same transformations but in the reverse order.

a) Compress horizontally by a factor of$1/2$.

b) Translate 4 units to the right.

Write the equation of the transformed function, and sketch its graph on the same grid below.



Equation of the transformed function:

$(-1,1) \rightarrow (-.5,1) \rightarrow (3.5,1)$

$(0,0) \rightarrow (0,0) \rightarrow (4,0)$

$(1,1) \rightarrow (.5,1) \rightarrow (4.5,1)$

$y=f(x) \rightarrow y=f(2x) \rightarrow y=f(2(x-4))$

$y=|x| \rightarrow y=|2x| \rightarrow y=|2(x-4)|$

$x \rightarrow 2x$ $\qquad$ $x \rightarrow x-4$

What can you conclude about the order of applying a horizontal translation and a horizontal compression or expansion? the order does matter.

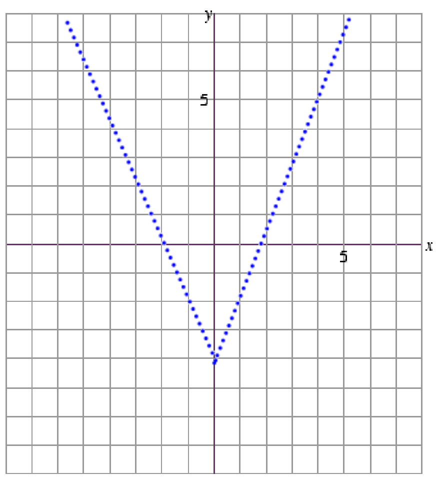
Part 3

Sketch the graph of $y=|x|$ on the grid below. Then, perform the following transformations on the function.

a) Translate 4 units down.

b) Compress horizontally by a factor of $1/2$.

Write the equation of the transformed function, and sketch its graph on the same grid below.



$(-2,2) \to (-2,-2) \to (-1,-2)$

$(0,0) \to (0,-4) \to (0,-4)$

$(2,2) \to (2,-2) \to (1,-2)$

Equation of the transformed function:

$y=f(x) \longrightarrow y=f(x)-4 \longrightarrow y=f(2x)-4$

$y=|x| \longrightarrow y=|x|-4 \longrightarrow y=|2x|-4$

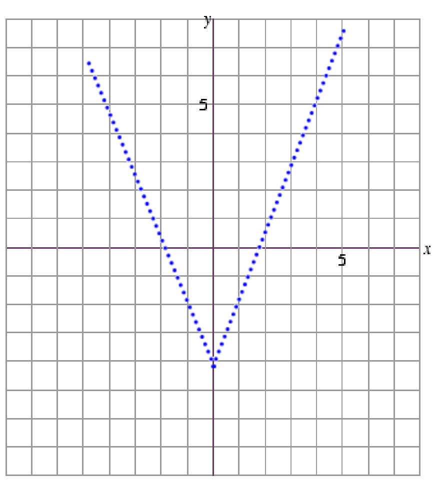
Part 4

As in Part 3, sketch the graph of $y=|x|$ on the grid below. This time, however, perform the same transformations in the reverse order.

a) Compress horizontally by a factor of $1/2$.

b) Translate 4 units down.

Write the equation of the newly transformed function, and sketch its graph on the same grid below.



Equation of the transformed function:

$(-2,2) \to (-1,2) \to (-1,-2)$

$(0,0) \to (0,2) \to (0,-2)$

$(2,2) \to (1,2) \to (1,-2)$

$y=f(x) \to y=f(2x) \to y=f(2x)-4$

$y=|x| \to y=|2x| \to y=|2x|-4$

What can you conclude about the order of applying a vertical translation and a horizontal compression or expansion?

Order does <u>not</u> matter.

Example 1:
Describe what happens to the equation of a function $y = f(x)$ when you expand its graph vertically by a factor of 2, then translate 3 units up.

$y=f(x) \longrightarrow y=2f(x) \longrightarrow y=2f(x)+3$

Describe what happens to the equation of a function $y = f(x)$ when you translate 3 units up, then expand its graph vertically by a factor of 2.

$y=f(x) \longrightarrow y=f(x)+3 \longrightarrow y=2[f(x)+3]$ or $y=2f(x)+6$

Describe what happens to the equation of a function $y = f(x)$ when you reflect in the y-axis, expand its graph vertically by a factor of 2, translate 3 units up, compress horizontally by a factor of ½ and finally translate 9 units left.

order matters.

Transformation	Equation
Original function	$y = f(x)$
reflect in the y-axis	$y=f(-x)$
expand vertically by a factor of 2	$y=2\cdot f(-x)$
translate 3 units up	$y=2f(-x)+3$
compress horizontally by a factor of ½	$y=2f(-2x)+3$
translate 9 units left	$y=2f(-2(x+9))+3$

Example 2: Given the equations, complete the following table by describing the transformation that has occurred at each stage of the mapping.

Transformation	Equation
Original function	$y = x^3$
v. reflection in x-axis.	$y = -x^3$
v. expansion by factor of 4	$y = -4x^3$
h. compression by factor of $\frac{1}{3}$	$y = -4[3x]^3$
h. translation 1 unit right.	$y = -4[3(x-1)]^3$
v. translation 5 units down.	$y = -4[3(x-1)]^3 - 5$

Example 3:
The function $y = f(x)$ is transformed to $y = f(2x+4)$. Identify the horizontal expansion or compression factor, and then describe the following translation that occurs.

$y = f((2x)+4)$

$y=f(2x+4)$ or $y=f(2(x+2))$

$y=f(x) \rightarrow y=f(x+4) \rightarrow y=f(2x+4)$

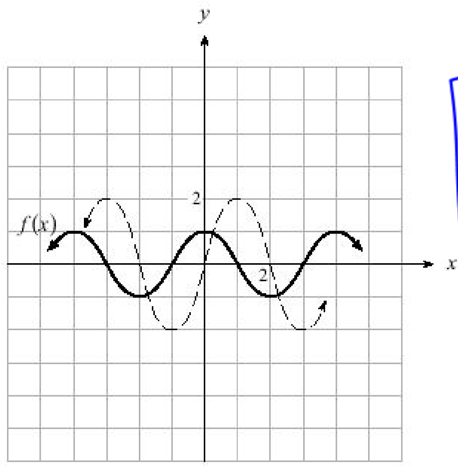
this shows translation first, then compression

$\boxed{\text{compression by factor of } \tfrac{1}{2} \text{ then translation 2 units left}}$

Example 4:
In the diagram below, $y = f(x)$ is graphed as a solid line. Write the equation of the function defined by the broken line.

v. expansion by factor of 2.

h. translated 1 unit right.



$\boxed{y = 2f(x-1)}$

* shortcuts

Example 5

The point $(-4, 6)$ is on the graph of $y = f(x)$. Determine the coordinates of the point on the transformed function:

a) $y = -\underbrace{f(2(x+1))}-5$

$y_n = -(6) - 5$

$y_n = -11$

$x_n = -3$

$(-3, -11)$

shortcut to find x,

x_0 = whatever is inside function

b) $y = -3\underbrace{f(-2x+10)}+8$

$y_n = -3(6) + 8$

$y_n = -10$

$x_n = 7$

$(7, -10)$

$-4 \to -14 \to$

$\boxed{x \to x+10}$ $\boxed{x \to -2x}$

c) $\dfrac{-y-3}{5} = f(-2x+8)$

$-y-3 = 5f(-2x+8)$

$-y = 5f(-2x+8) + 3$

$y = -5f(-2x+8) - 3$

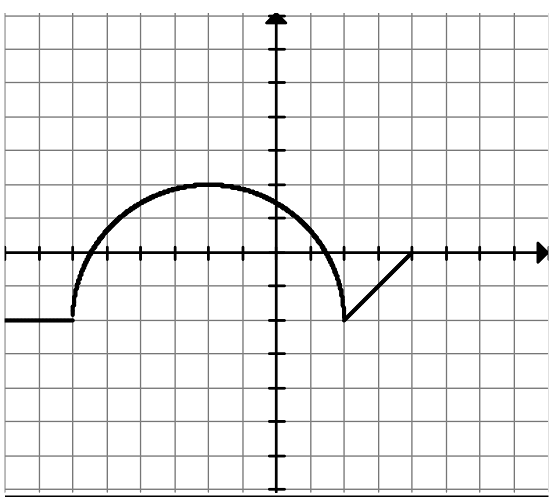
$y_n = -33$

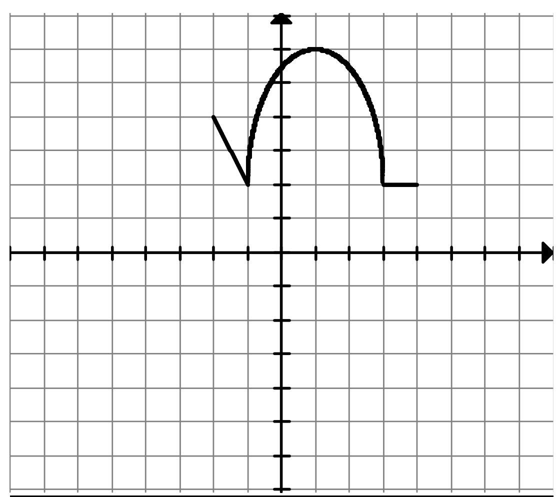
$x_n = 6$

$\boxed{(6, -33)}$

Example 6

Given the graph of $y = f(x)$ below, determine the equation of the transformed graph to the right.





p38 #1-8

p 38 #9-11, 13, 17, C2, C3 * 16, 18.