Warmup

Determine the derivatives of

1. $y = \pi^x$

2. $y = x^\pi$

3. $y = 5^{\sin x^2}$

4. A tangent to the graph of $y = 2^x$ passes through the origin. Find the equation of this tangent.

5. Find all points on the graph $y = (x+1)e^x$ where there is a horizontal tangent.

Derivatives of Logarithmic Functions

To find the derivative of $y = \ln x$, we use the fact that $y = \ln x \iff x = e^y$ (These are two different ways of stating the same thing)

Thus we can rewrite $y = \ln x$ as $x = e^y$ and differentiate implicitly.

*

$x = e^y$

$1 = e^y \cdot y'$

$y' = \frac{1}{e^y}$

$\frac{d(\ln x)}{dx} = \frac{1}{x}$

Handy Dandy Logarithmic Properties

$$a = e^{\ln a} = B^{\log_B a}$$

$$a^x = e^{x \ln a} = B^{x \log_B a}$$

*Base change formula.

$$\log_B x = \frac{\ln x}{\ln B} = \frac{\log x}{\log B} = \frac{\log_A x}{\log_A B}$$

$\ln x = \log_e x$

$$\frac{d}{dx} \ln u = \frac{1}{u} \cdot \frac{du}{dx}$$

To differentiate $y = \log_B x$, use the change of base formula and then differentiate.

$y = \frac{\ln x}{\ln B}$

$y = \frac{1}{\ln B} \cdot \ln x$

$y' = \frac{1}{\ln B} \cdot \frac{1}{x}$

$$\frac{d}{dx} \log_B u = \frac{1}{\ln B} \cdot \frac{1}{u} \cdot \frac{du}{dx}$$

Determine the derivatives of the following functions

1. $y=\ln 8x$

$$y' = \frac{1}{8x}\cdot 8$$
$$y' = \frac{1}{x}$$

2. $y=\ln x^4$

$$y' = \frac{1}{x^4}\cdot 4x^3$$
$$y' = \frac{4}{x}$$

* be aware you may need
a) implicit differentiation
b) rules for logarithms.

3. $y=\ln(\cos x)$

$$y' = \frac{1}{\cos x}\cdot -\sin x$$
$$y' = -\tan x$$

4. $y=\ln\left(x^2\sqrt{x+1}\right)$ $\quad x^2\cdot(x+1)^{\frac{1}{2}}$

$$y' = \frac{1}{x^2\cdot\sqrt{x+1}}\left(2x\sqrt{x+1}+\tfrac{1}{2}(x+1)^{-\frac{1}{2}}\cdot x^2\right)$$
$$y' = \frac{1}{x^2\sqrt{x+1}}\left(\frac{2x\sqrt{x+1}}{1}+\frac{x^2}{2\sqrt{x+1}}\right)$$
$$y' = \frac{2}{x}+\frac{1}{2(x+1)}$$

or

$$y = \ln x^2 + \ln\sqrt{x+1}$$
$$y' = \frac{1}{x^2}\cdot 2x + \frac{1}{\sqrt{x+1}}\cdot\frac{1}{2}(x+1)^{-\frac{1}{2}}$$
$$y' = \frac{2}{x}+\frac{1}{2(\sqrt{x+1})^2}$$
$$y' = \frac{2}{x}+\frac{1}{2(x+1)}$$

5. $y=\log_{10} x^8$

$$y' = \frac{1}{\ln 10}\cdot\frac{1}{x^8}\cdot 8x^7$$

6. $y=\log_5\left(1+x^3\right)$

$$y' = \frac{1}{\ln 5}\,\frac{1}{1+x^3}\cdot 3x^2$$

use log rule for quotients.

7. $y = \log_2\left(\frac{x^2}{x^3-8}\right)$

$y = \log_2 x^2 - \log_2(x^3-8)$

$y' = \frac{1}{\ln 2} \cdot \frac{1}{x^2} \cdot 2x - \frac{1}{\ln 2} \cdot \frac{1}{(x^3-8)} \cdot 3x^2$

$$\boxed{y' = \frac{1}{\ln 2}\left(\frac{2}{x} - \frac{3x^2}{x^3-8}\right)}$$

8. Show that the Power Rule $\frac{d}{dx}x^n = n \bullet x^{n-1}$ is true for all real n.

9. $y = (x + \sin 2x)^{\sqrt{2}}$

$y' = \sqrt{2}\,(x + \sin 2x)^{\sqrt{2}-1} \cdot [1 + \cos(2x) \cdot 2]$

10. $y = x^x$

* neither exponential functions nor power functions

* use implicit differentiation

$\ln y = \ln x^x$

$\ln y = x \cdot \ln x \longrightarrow \frac{1}{y} \cdot y' = 1 \cdot \ln x + \frac{1}{x} \cdot x$

$y' = y(\ln x + 1)$

$$\boxed{y' = x^x(\ln x + 1)}$$

11. $y = x^{\sin x}$

$\ln y = \ln(x^{\sin x})$

$\ln y = (\sin x)(\ln x)$

$\frac{1}{y} \cdot y' = \cos x \ln x + \frac{1}{x} \cdot \sin x$

$y' = y(\cos x \ln x + \frac{1}{x} \sin x)$

$y' = x^{\sin x}\left(\cos x \cdot \ln x + \frac{\sin x}{x}\right)$