Derivatives of Exponential Functions

Calculus 12 Name________________________
Unit 3.5

Recognize the difference between exponential functions and power functions

Exponential Functions	**Power Functions**
Variable in the exponent, base is constant	Variable in the base, exponent is constant
Ex: $y=2^x$, $y=e^{\sin x}$, $y=\left(\frac{1}{2}\right)^{x^2+7x+1}$	Ex: $y=x^4$, $y=(\sin x)^8$, $y=\left(x^2+\cos x\right)^{-\frac{2}{3}}$ $y=x^e$ $y=x^{\sqrt{2}}$
Rule: $\frac{d}{dx}a^u=a^u \bullet \ln a \bullet \frac{du}{dx}$	Rule: $\frac{d}{dx}u^n=n\,u^{n-1}\frac{du}{dx}$

1. Determine the derivatives of each of the following functions:

a) $y=5^{\sin x}$ $y'=5^{\sin x}\cdot\ln 5\cdot(\cos x)$

b) $y=e^{2x+3}$ $\boxed{y'=e^{2x+3}\cdot\underbrace{\ln e}_{\text{cancel out}}\cdot(2) = 2\cdot e^{2x+3}}$

c) $y=7^{\sin x+\cos x}$ $\boxed{y'=7^{\sin x+\cos x}\cdot\ln 7\cdot(\cos x-\sin x)}$

d) $y=e^2x+2x^e$ $y'=e^2+2ex^{e-1}$

e) $y=10^x+x^{10}$ $y'=10^x\cdot\ln 10\cdot(1)+10x^9$
$\boxed{y'=10^x\cdot\ln 10+10x^9}$

f) $y=x^{\sqrt{2}}$ $y'=\sqrt{2}\cdot x^{(\sqrt{2}-1)}$

g) $y=\sqrt{2}^x$ $y'=\sqrt{2}^x\cdot\ln\sqrt{2}\cdot 1$

2. Find the equation of the tangent to $y = 2^x$ at $x = 1$.

coordinate $y(1) = 2^1$

$y = 2$

$(1, 2)$

$$\boxed{y - 2 = 2\ln 2(x-1)}$$

$m_{tan} = y'\big|_{x=1}$

$= 2^x \cdot \ln 2 \cdot (1)\big|_{x=1} = 2\ln 2$

3. Find all points on the curve $y = x^2 e^x$ where the tangent line is horizontal.

product rule

$\underline{\underline{or}}$ $m_{tan} = 0$

$y' = 2x \cdot e^x + e^x \cdot \underline{\ln e \cdot 1} \cdot x^2$

$0 = 2x \cdot e^x + x^2 e^x$

$0 = e^x \cdot x(2 + x)$

$x = 0, -2$

$\underline{\underline{x = 0}}$

$y = 0^2 \cdot e^0$

$y = 0$

$\underline{\underline{(0, 0)}}$

$\underline{\underline{x = -2}}$

$y = (-2)^2 \cdot e^{-2}$

$= 4 \cdot \frac{1}{e^2}$

$\underline{\underline{\left(-2, \frac{4}{e^2}\right)}}$

4. Determine $\lim\limits_{h \to 0} \dfrac{5^{x+h} - 5^x}{h}$

$$= \frac{d(5^x)}{dx} = \boxed{5^x \cdot \ln 5}$$

5. Determine the equation of the tangent to $y = e^x$ which passes through the origin.

$m_{tan} = \dfrac{y - 0}{x - 0}$ $\big\}$ $y = e^x$

$m_{tan} = \dfrac{e^x - 0}{x - 0}$

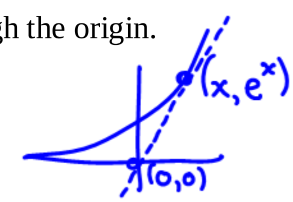
$e^x = \dfrac{e^x}{x}$

$y' = e^x$

$y' = e^x\big|_{x=1}$

$\underline{\underline{x = 1}}$

$y = e^1$



$$\boxed{y - e = e^1(x - 1)}$$