The Derivative of $y = e^x$

By the definition of the derivative, $f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$, we have that if $f(x) = e^x$ then

$$
\begin{aligned}
f'(x) &= \lim_{h \to 0} \frac{e^{x+h} - e^x}{h} \quad \leftarrow \text{exponent laws: } e^{x+h} = e^x \cdot e^h \\
&= \lim_{h \to 0} \frac{e^x \cdot e^h - e^x}{h} \\
&= \lim_{h \to 0} \frac{e^x \cdot \left(e^h - 1\right)}{h} \\
&= \underbrace{\lim_{h \to 0} e^x}_{e^x} \bullet \lim_{h \to 0} \frac{e^h - 1}{h}
\end{aligned}
$$

This leaves us with the problem of determining $\lim\limits_{h \to 0} \dfrac{e^h - 1}{h}$

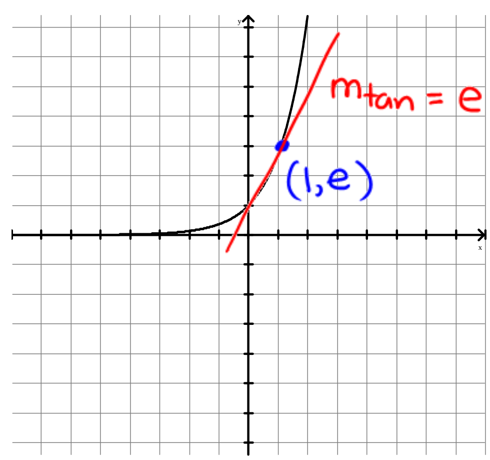
By examining this limit numerically, we have the following:

h	$\dfrac{e^h - 1}{h}$	h	$\dfrac{e^h - 1}{h}$
0.1	1.1052	–0.1	0.9048
0.01	1.0101	–0.01	0.9900
0.001	1.0010	–0.001	0.9990
0.0001	1.0001	–0.0001	0.9999

Thus it appears that $\lim\limits_{h \to 0} \dfrac{e^h - 1}{h} = \underline{\quad 1 \quad}$

Applying this to the limit above, we have $\dfrac{d}{dx} e^x = \lim\limits_{h \to 0} e^x \bullet \lim\limits_{h \to 0} \dfrac{e^h - 1}{h} = \lim\limits_{h \to 0} e^x \bullet 1 = \underline{\quad e^x \quad}$

In other words, $\boxed{\dfrac{d}{dx} e^x = e^x}$ or the derivative of the exponential function e^x is simply the function itself. This is the **only** function (other than the trivial function $\underline{y = 0}$) which possesses this property. Put in another way, on the graph of $y = e^x$, the *y* coordinate of any point also tells you the slope at that point.



$y = (2)^x$

$2 = e^{\square}$

$\cancel{\log_2} 2^4 = 4$

$\ln 2 = \cancel{\ln} e^{\square}$

$\ln 2 = \square$

$y = (e^{\ln 2})^x$

$\boxed{2 = e^{\ln 2}}$

$y = \boxed{e^{x \ln 2}}$

$y' = e^{x \cdot \ln 2} \cdot \ln 2$

$$\frac{d(2^x)}{dx} = 2^x \cdot \ln 2$$