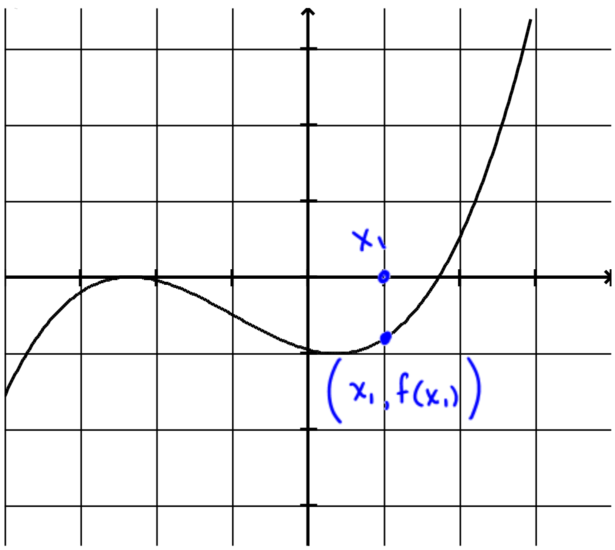
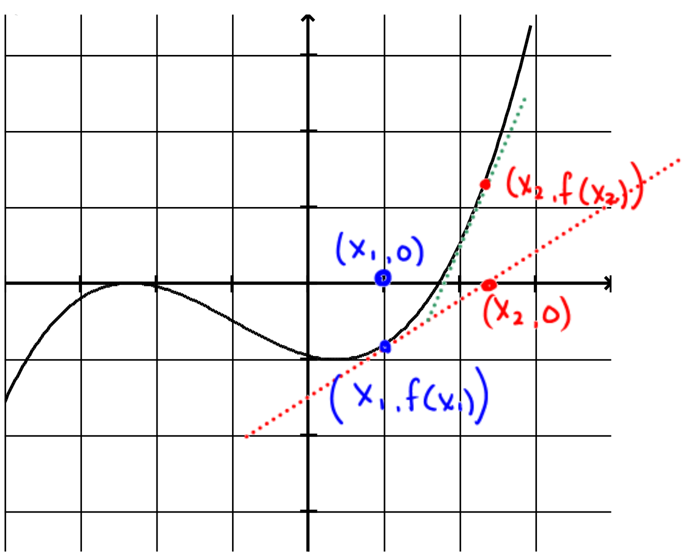
Newton's Method
A Method for Finding Zeros

We want to solve the equation $f(x)=0$ where *f* is differentiable over an open interval containing the root(s). Let *r* be a root, or solution, of $f(x)=0$; that is $f(r)=0$. Newton's Method is an ***iterative*** process; i.e. a successive repetition of a mathematical process using the result of one stage as the input for the next. Each calculation which gets closer to the real value is called an ***iteration***. We proceed as follows:



1) Consider an initial estimate x_1 that is "close" to r. To get an initial estimate, we might locate possible roots between integers by looking for sign changes, guess and check or perhaps sketch.

2) Determine a second approximation by drawing the tangent to the curve at x_1. Determine the *x*-intercept of the tangent, and use this as your next approximation. Repeat this process of going to the curve, drawing the tangent and finding the *x*-intercept to use as your next approximation



This process yields in general the following

$$f'(x_1) = \frac{0-f(x_1)}{x_2-x_1}$$

$$x_2 - x_1 = \frac{-f(x_1)}{f'(x_1)}$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \qquad n = 1, 2, 3, \ldots$$

If $|x_{n+1} - x_n|$ is less than the desired accuracy, let x_{n+1} serve as the final approximation. Otherwise, continue step 2 until x_{n+1} and x_n agree to the correct number of decimal places.

1. Starting with $x_1 = -1.2$, find the third approximation x_3 to the root of $f(x) = 2x^3 + x^2 - x + 1$

Y_1

$f' = 6x^2 + 2x - 1$

Y_2

$$X_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= -1.2 - \frac{f(-1.2)}{f'(-1.2)}$$

$$X_2 = -1.235114504$$

$$X_3 = -1.235114504 - \frac{f(-1.235114504)}{f'(-1.235114504)}$$

$$X_3 = -1.233754022$$

On graphing calculator

$Y_1 = f(x)$ $\qquad\qquad$ $Y_2 = f'(x)$

Then store first guess in x or in ANS. Then program your calculator to do Newton's Method:

$x - \dfrac{Y_1(x)}{Y_2(x)} \rightarrow x$ or $Ans - \dfrac{f(Ans)}{f'(Ans)}$ (This technique can be used on most Scientific Calculators)

Each time ENTER is pressed, another iteration is performed.

2. Find the point of intersection of $y = x^3$ with $y = 1 - x$ correct to 6 decimal places.

$f(0) = -1$

$f(1) = 1$

$f(2) = 9$

there is an x-int between 0 and 1

$$x^3 = 1 - x$$

$$x^3 + x - 1 = 0$$

$$X_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$f(x) = x^3 + x - 1 \Rightarrow Y_1$

$f'(x) = 3x^2 + 1 \Rightarrow Y_2$

$X_1 = 0.1$

$X_2 = 0.972815534$

$X_3 = 0.7400899835$

$X_4 = 0.6850575035$

$X_5 = 0.6823341551$

$X_6 = 0.682328$

3. Let $f(x) = e^{x-2} + x^3 - 2$

a) Use the derivative of f to explain why the equation $f(x) = 0$ has at most one solution.

b) Explain why $f(x) = 0$ has a solution in the interval $(1, 2)$

c) Newton's Method with an initial estimate of 2 is used to find an approximate value for the solution of $f(x) = 0$. What is the next estimate

a) $f' = e^{x-2} + 3x^2$

$\oplus + \oplus$

f' is always $\oplus$

$\therefore f(x)$ is always increasing

$\therefore$ only 1 x-int.

b) $f(1)$ = negative.

$f(2)$ = positive

$\therefore$ must be at least x-int in this function