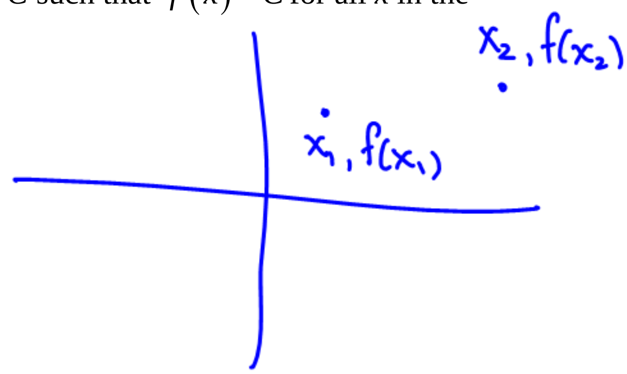
Consequences of Mean Value Theorem

1. Functions with $f' = 0$ for all x are constant

If $f'(x) = 0$ for all x in some interval I, then there exists a constant C such that $f(x) = C$ for all x in the interval I

Proof:

$$f'(x) = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$



$$0 = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$\therefore$ $f(x_2) = f(x_1)$ for all values of x; the graph is a horzontal line / constant function

2. Functions with the same derivative differ by a constant

If $\boxed{f'(x) = g'(x)}$ for all x in some interval I, then $f(x) = g(x) + C$ for all x in I

Proof:

$$f(x) = g(x) + h(x)$$

$$f'(x) = g'(x) + h'(x)$$

$$f'(x) = f'(x) + h'(x)$$

$$0 = h'(x)$$

if $h'(x) = 0$ then $h(x)$ is a constant

$$f(x) = g(x) + C$$

Example: Find the function whose derivative is $\sec^2 x$ and which passes through $(\pi, 5)$

$$f'(x) = \sec^2 x$$

$$f(x) = \tan(x) + C$$

$$5 = \tan(\pi) + C$$

$$5 = 0 + C$$

$$C = 5$$

$$\boxed{f(x) = \tan(x) + 5}$$

The Antiderivative

The function $F(x)$ is an **antiderivative** of $f(x)$ if $\underline{F'(x) = f(x)}$ for all x in the domain of f. The process of finding the antiderivative is called **antidifferentiation**.

Example 2: Find the antiderivative of the following functions

a) $f'(x) = k x^0$ b) $f'(x) = x^1$ c) $f'(x) = x^2$ d) $f'(x) = x^3$ e) $f'(x) = x^n$

$f(x) = kx + C$

$f(x) = \frac{1}{2}x^2 + C$

$f(x) = \frac{1}{3}x^3 + C$

$f(x) = \frac{1}{4}x^4 + C$

$f(x) = \frac{1}{n+1}x^{n+1} + C$

Example 3: Find the velocity and position functions when an object falls from rest and the acceleration is $9.8\ \text{m}/\text{sec}^2$

initial velocity $= 0$

initial height $= h_0$

$s(x) =$ position function

$v(x) = s'(x) =$ velocity function

$a(x) = v'(x) = s''(x) =$ acceleration function.

$a(x) = -9.8\ m/s^2$

$v(x) = -9.8x + C$

$v(0) = 0$

$0 = -9.8(0) + C$

$C = 0$

$\boxed{\therefore\ v(x) = -9.8x}$

$s(x) =$ antiderivative of $v(x)$

$s(x) = -9.8 \cdot \frac{1}{2} \cdot x^2 + C$

$s(x) = -4.9x^2 + C$

$h_0 = -4.9(0)^2 + C$

$C = h_0$

$\boxed{s(x) = -4.9x^2 + h_0}$

Example 4: Find the velocity and position functions when an object is thrown upward with initial velocity v_0 and the acceleration is a (where v_0 and a are constants)

$v(0) = V_0$

$a(x) = a$

$v(x)$ is antiderivative of $a(x)$

$v(x) = ax + C$

$V_0 = a(0) + C$

$C = V_0$

$\boxed{v(x) = ax + V_0}$

s(x) is antiderivative of v(x)

$$s(x) = a \cdot \left(\tfrac{1}{2}\right) x^2 + V_0 x + C$$

$h(0) = h_0$

$$h_0 = \tfrac{1}{2} a (0)^2 + V_0 (0) + C$$

$$h_0 = C$$

$$\boxed{s(x) = \tfrac{1}{2} a x^2 + V_0 x + h_0}$$