

## 5.2 Extra Practice

1. What is the greatest common factor (GCF) of each set of numbers?
    - 28 and 98
    - 243 and 162
    - 192 and 216
    - 90, 105, and 165
    - 48, 120, and 168
  2. Determine the least common multiple (LCM) of each set of numbers.
    - 12 and 26
    - 9 and 36
    - 6 and 15
    - 4, 5, and 12
    - 16, 20, and 44
  3. Determine the GCF of each set of terms.
    - $15x^4$  and  $5x^2y$
    - $-24xy$  and  $8xy$
    - $ax^2$  and  $-bx$
    - $18y^4$ ,  $-9y^3$ , and  $-27y^2$
    - $2\pi xr$ ,  $-2\pi xr$ , and  $2\pi xh$
  4. Factor each polynomial, if possible.
    - $5x + 35$
    - $4x + 23$
    - $14x - 8y$
    - $6x^2 + 24x$
    - $3x + 9xy + 6xz$
  5. Identify each missing factor.
    - $3ax + 3ay = (\boxed{\quad})(x+y)$
    - $x^2 - xy = (x)(\boxed{\quad})$
    - $25ab - 10ab^2 = (5ab)(\boxed{\quad})$
    - $6x^2 - 3x^3 - 9x = (\boxed{\quad})(2x - x^2 - 3)$
    - $3x^3 - x^2y + 6xy^2 = (x)(\boxed{\quad})$
6. Factor each polynomial.
- $8x^2 + 32y^3$
  - $10a + 5a^2 - 25a^3$
  - $24abc - 6ab + 8bc$
  - $-12x^2y^2 + 3xy^3 - 15x^3y$
  - $9\pi x^2 - 6xy + 12\pi xy^2$
7. Write each expression in factored form, if possible.
- $x(y+1) + 4(y+1)$
  - $3x(a+b) - y(a+b)$
  - $4y(y+3) + (y+3)$
  - $5a(2x+1) + 3(2x-1)$
  - $3y(x-5) - 4(5-x)$
8. Factor by grouping.
- $5x + 15y + mx + 3my$
  - $xy + 4x + 5y + 20$
  - $3ab - 3ac + 2b^2 - 2bc$
  - $-5y + 3 - 6x + 10xy$
  - $2x^2 + xz + 6xy + 3yz$
9. Write an expression in factored form to represent the area of each shaded region.
- a)
- 
- b)
- 

<b>5.2 Extra Practice Key</b>	<b>1. a) 14 b) 81 c) 24 d) 15 e) 24</b>	<b>2. a) 156 b) 36 c) 30 d) 60 e) 880</b>	<b>3. a) <math>5x^2</math> b) <math>8xy</math> c) <math>x</math> d) <math>9y^2</math> e) <math>2\pi x</math></b>	
<b>4. a) <math>5(x+7)</math> b) not possible c) <math>2(7x-4y)</math> d) <math>6x(x+4)</math> e) <math>3x(1+3y+2z)</math></b>	<b>5. a) <math>\square = 3a</math> b) <math>\square = x-y</math> c) <math>\square = 5-2b</math> d) <math>\square = 3x</math></b>			
<b>e) <math>\square = 3x^2 - xy + 6y^2</math> 6. a) <math>8(x^2 + 4y^3)</math> b) <math>5a(2+a-5a^2)</math> c) <math>2b(12ac-3a+4c)</math> d) <math>3xy(-4xy+y^2-5x^2)</math></b>	<b>7. a) <math>(y+1)(x+4)</math> b) <math>(a+b)(3x-y)</math> c) <math>(y+3)(4y+1)</math> d) not possible e) <math>(x-5)(3y+4)</math></b>			
<b>8. a) <math>(x+3y)(5+m)</math> b) <math>(y+4)(x+5)</math> c) <math>(b-c)(3a+2b)</math> d) <math>(5y-3)(2x-1)</math> e) <math>(2x+z)(x+3y)</math></b>	<b>9. a) <math>3x(5x+2y)</math> b) <math>2x(2\pi x-3y)</math></b>			

### 5.3 Warm-Up

1. Expand.

a)  $(3x - 5)(x + 4)$

b)  $(x + 4y)(2x - 5y)$

2. Factor out the greatest common factor.

a)  $3x^2 + 9x$

b)  $8xy - 6y^2$

3. Factor by grouping.

a)  $x(x - 5) + 2(x - 5)$

b)  $2x(x + 2y) + 5y(x + 2y)$

4. Write all the pairs of integers that multiply to

a) 12

b) 7

c) -7

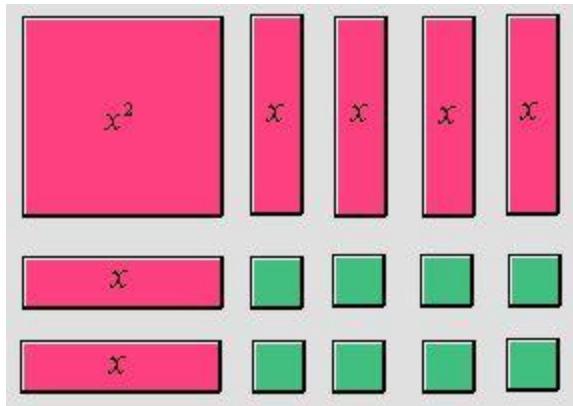
5. a) Write all the pairs of integers that multiply to -6.

b) Which pair in part a) adds to 1?

c) Which pair adds to -5?

### 5.3a Factoring Trinomials of the form $x^2 + bx + c$

What Trinomial is represented by these algebra tiles? \_\_\_\_\_



Tiles placed along the top side and left side, represent the factors: \_\_\_\_\_ and \_\_\_\_\_.

We can check our conclusion by multiplying the two binomials.

Thus, the trinomial \_\_\_\_\_ is a product of \_\_\_\_\_ and \_\_\_\_\_.

Model the trinomials  $x^2 + 5x + 6$  and  $x^2 + 3x + 2$  with algebra tiles. What are the binomial factors that multiply to give each as a product?

$$x^2 + 5x + 6$$

$$x^2 + 3x + 2$$

We can also factor without the use of manipulatives. When trinomials are of the form  $x^2 + bx + c$ , a pattern forms between the  $b$  term and the  $c$  term.

Trinomial	Binomial Factor	Binomial Factor
$x^2 + 5x + 6$	$x + 3$	$x + 2$
$x^2 + 8x + 12$	$x + 2$	$x + 6$
$x^2 + 3xy - 18y^2$	$x + 6y$	$x - 3y$
$x^2 + 4x + 6$	Cannot be factored	

What patterns do you notice in the table above?

We can factor by listing the factors of the  $c$  term and then choosing the two which ADD to give the  $b$  term.

Factor:  $x^2 + 11x + 24$  Factors of 24: \_\_\_\_\_

Two factors that add/subtract to +11: \_\_\_\_\_ and \_\_\_\_\_

$x^2 + 12x + 20$	$n^2 + 5n + 6$	$n^2 - 5n - 24$	$p^2 + p - 90$
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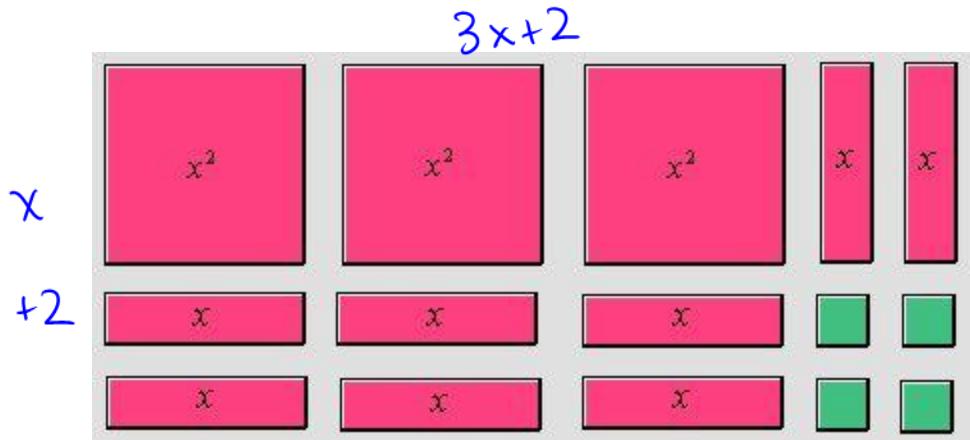
REMEMBER! Always look for a \_\_\_\_\_ first!

$$4x^2 + 12x + 8$$

$2x^2 + 8x + 6$ GCF:	$-4x^2 - 4x + 48$ GCF:	$x^4 + 8x^3 + 12x^2$ GCF:
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### 5.3 b Factoring Trinomials of the form $ax^2 + bx + c$

What Trinomial is represented by these algebra tiles?  $3x^2+8x+4$



Tiles placed along the top side and left side, represent the

factors:  $(3x+2)$  and  $(x+2)$ .

We can check our conclusion by multiplying the two binomials.

$$(3x+2)(x+2) = 3x^2 + 6x + 2x + 4$$

Thus, the trinomial  $3x^2+8x+4$  is a product of  $3x+2$  and  $x+2$ .

Model the trinomial  $2x^2 + 7x + 6$  with algebra tiles. What are the binomial factors that multiply to give this product?

\* Factoring by decomposition

$$\overline{2x^2} \stackrel{12}{=} \overline{+7x} \overline{+6}$$

$$2x^2 + 4x + 3x + 6$$

$$2x(x+2) + 3(x+2)$$

$$(x+2)(2x+3)$$

$$\begin{array}{r} 2x^2 + 3x + 4x + 6 \\ \hline x(2x+3) + 2(2x+3) \end{array}$$

$$(2x+3)(x+2)$$

Factor each of the following. What is different about the two?

$$2x^2 + 8x + 6$$

$$2x^2 + 2x + 6x + 6$$

$$2x(x+1) + 6(x+1)$$

$$(2x+6)(x+1)$$

GCF

$$\boxed{2(x+3)(x+1)}$$

completely factored

$$2x^2 + 5x + 3$$

$$2x^2 + 3x + \underline{2x+3}$$

$$x(2x+3) + 1(2x+3)$$

$$(2x+3)(x+1)$$

\* Sometimes you will have a GCF of 1

Sometimes a trinomial will have a leading coefficient that is not 1 and there is no greatest common factor. Without using algebra tiles, we factor by a different method:

$$2x^2 + 5x + 3$$

a) Rectangle Method  
- basically same as decomposition

	x	1
2x	$2x^2$	$2x$
3	$3x$	3

$$(2x+3)(x+1)$$

1. Remove GCF if possible
2. Identify 2 numbers that multiply to make  $ac$  and add to make  $b$ .
3. Find the last terms as for normal factoring.  
The first terms are the same as the original trinomial

New steps:

4. Divide the factored polynomial by the \_\_\_\_\_ of the \_\_\_\_\_ term.

5. Remove any GCF's and cancel factors.

$$\frac{(2x+3)(2x+2)}{2}$$

$$(2x+3)(x+1)$$

b) decomposition

$$\underbrace{2x^2 + 2x}_{2x(x+1)} + \underbrace{3x + 3}_{3(x+1)}$$

$$2x(x+1) + 3(x+1)$$

$$(2x+3)(x+1)$$

c) Algorithm method

$$\overline{2x^2 + 5x + 3} \quad 6$$

When can a trinomial be factored?

$$x^2 + 9x + 10$$

$3x^2 + 10x + 3$  can be factored  $9, 1$

$6x^2 - 5x + 3$  no

no nothing that multiplies to make 10 and adds to make 9

For what values of  $k$  can  $4x^2 + kx + 2$  be factored?

$$4x^2 + \underline{-}x + 2$$

8

$$\begin{array}{l} 1, 8 \\ 2, 4 \\ -2, -4 \\ -1, -8 \end{array}$$

$$\begin{array}{l} k \text{ could be } \pm 9, \pm 6 \\ k = \{\pm 9, \pm 6\} \end{array}$$

Factor each of the following completely, if possible:

$2x^2 + 7x + 6$ <u>12</u> $\underbrace{2x^2 + 4x}_{2x(x+2)} + \underbrace{3x + 6}_{3(x+2)}$ $(2x+3)(x+2)$	$8x^2 + 4x + 4$ <u>32</u> can't be factored	$3x^2 - 22x + 7$ <u>21</u> $(3x-1)(x-7)$
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$5x^2 - 7x + 2$	$6a^2 + 11a + 3$	$14c^2 - 6c - 7c + 3$
$(5x-2)(x-1)$	$3(2a+3) + 1(2a+3)$ $(3a+1)(2a+3)$	$2c(7c-3) + (-1)(7c-3)$ $(2c-1)(7c-3)$

### Applications

A rescue worker launches a signal flare into the air from the side of a mountain. The height of the flare is represented by the formula  $h = -16t^2 + 144t + 160$ . h is height (in feet) and t is time (in seconds).

- a) What is the factored form of the formula? b) What is the height after 5.6 s?

look for GCF first

$$h = -16(t^2 - 9t - 10)$$

$$h(t) = -16(t+1)(t-10)$$

b) substitute  $t = 5.6$

$$\begin{aligned} h(5.6) &= -16(5.6+1)(5.6-10) \\ &= 464.64 \text{ feet} \end{aligned}$$

$$4x^2 + 11x + k \quad 1 < k < 20$$

$$K=6 \text{ or } K=7$$

$$\begin{matrix} 10,1 \\ 9,2 \\ 8,3 \\ 7,4 \\ 5,6 \end{matrix}$$

$$\begin{aligned} 4 \times k &= 24 & K &= 6 \\ 4 \times k &= 28 & K &= 7 \end{aligned}$$

5.3 Extrapractice #6-9

P 234 #2, 3, 6, 7, #8b, 11, 12, 15, 18, 19

## 5.3 Extra Practice

1. Identify two integers with the given product and sum.

- a) product = 12, sum = 13
- b) product = 34, sum = 19
- c) product = -33, sum = 8
- d) product = -20, sum = -1
- e) product = 54, sum = -15

2. Factor, if possible.

- a)  $x^2 + 8x + 15$
- b)  $x^2 + 5x + 6$
- c)  $x^2 + 11x + 28$
- d)  $m^2 + 7m + 10$
- e)  $y^2 + 24y + 144$

3. Factor, if possible.

- a)  $x^2 - 13x + 42$
- b)  $x^2 - 18x + 81$
- c)  $x^2 - x - 20$
- d)  $x^2 + 5x - 6$
- e)  $x^2 - x + 1$

4. Factor each trinomial.

- a)  $x^2 + 9xy + 14y^2$
- b)  $x^2 - 8xy + 16y^2$
- c)  $x^2 - 8xy + 15y^2$
- d)  $m^2 + 7mn - 8n^2$
- e)  $a^2 - 6ab - 7b^2$

5. Factor each trinomial. First check for a GCF.

- a)  $4x^2 + 24xy + 36y^2$
- b)  $2x^2 - 26x + 72$
- c)  $5x^2 - 5xy - 30y^2$
- d)  $-3x^2 - 48x - 165$
- e)  $3x^2 - 30x + 63$

6. Factor.

- a)  $2x^2 + 13x + 15$
- b)  $3x^2 + 11xy - 4y^2$
- c)  $7a^2 - 47a + 30$
- d)  $10y^2 + 9y + 2$
- e)  $12x^2 - 8x - 15$

7. Factor. First check for a GCF.

- a)  $12x^2 - 26x - 10$
- b)  $18x^2 - 3x - 36$
- c)  $75y^2 - 120y + 48$
- d)  $12x - 15xy - 18xy^2$
- e)  $40x^2y - 36xy^2 - 36y^3$

8. Determine two values of  $b$  so that each trinomial can be factored.

- a)  $x^2 + bx + 10$
- b)  $x^2 + bx + 8$
- c)  $x^2 - bx + 12$
- d)  $m^2 + 6m + b$
- e)  $y^2 + 5y + b$

9. Determine two values of  $k$  so that each trinomial can be factored.

- a)  $2x^2 + kx + 5$
- b)  $3x^2 + kx + 2$
- c)  $2x^2 + kx - 15$
- d)  $20m^2 + 23m + k$
- e)  $6y^2 + 17y + k$

### Section 5.3 Extra Practice

1. a) 1, 12 b) 2, 17 c) 11, -3 d) -5, 4 e) -6, -9
2. a)  $(x + 3)(x + 5)$  b)  $(x + 3)(x + 2)$   
c)  $(x + 4)(x + 7)$  d)  $(m + 2)(m + 5)$  e)  $(y + 12)^2$
3. a)  $(x - 6)(x - 7)$  b)  $(x - 9)^2$  c)  $(x + 4)(x - 5)$   
d)  $(x - 1)(x + 6)$  e) not possible
4. a)  $(x + 2y)(x + 7y)$  b)  $(x - 4y)^2$  c)  $(x - 3y)(x - 5y)$   
d)  $(m + 8n)(m - n)$  e)  $(a - 7b)(a + b)$
5. a)  $4(x + 3y)^2$  b)  $2(x - 4)(x - 9)$  c)  $5(x - 3y)(x + 2y)$   
d)  $-3(x + 11)(x + 5)$  e)  $3(x - 7)(x - 3)$
6. a)  $(2x + 3)(x + 5)$  b)  $(3x - y)(x + 4y)$  c)  $(7a - 5)(a - 6)$   
d)  $(2y + 1)(5y + 2)$  e)  $(2x - 3)(6x + 5)$
7. a)  $2(2x - 5)(3x + 1)$  b)  $3(3x + 4)(2x - 3)$  c)  $3(5y - 4)^2$   
d)  $3x(4 + 3y)(1 - 2y)$  e)  $4y(5x + 3y)(2x - 3y)$
8. Look for two values for each. a) 7, 11, -7, -11  
b) 6, 9, -6, -9 c) 7, 8, 13, -7, -8, -13 d) 5, 8, 9 e) 4, 6
9. Look for two values for each.  
a) 7, 11, -7, -11 b) 5, 7, -5, -7 c) 1, 7, 13, 29, -1, -7, -13, -29 d) 3, 6 e) 5, 7, 10, 11, 12

## 5.4: Factoring Special Trinomials

Find each of the following products

$$(x+5)(x-5)$$

$$(2x+3)(2x-3)$$

$$(2a-5)(2a+5)$$

In the questions above the polynomial product is only two terms instead of the usual three ( $ax^2 + bx + c$ )....what happened to the middle term ( $bx$ )?

These Binomial pairs are called \_\_\_\_\_.

If you recognize a polynomial as a difference of squares, then you can factor it a certain way.

$$16x^2 - 9$$

Factor each of the following:

$x^2 - 25$	$25x^2 - 4$	$x^2 - y^2$	$16 - a^2$
$9m^2 - 64$	$16x^2 - 49y^2$	$1 - 25x^2$	$m^2 + 1$

**Note:** A sum of squares cannot be factored.

REMEMBER! When factoring, you should always look for a \_\_\_\_\_ first.

After you take out your \_\_\_\_\_, sometimes your binomial can't  
\_\_\_\_\_ or sometimes you can do a second set of factoring!

Factor each of the following completely:

$$20x^2 - 5$$

$$8m^2 - 12$$

$$4x^2 - 4$$

Find each of the following products

$(x-5)(x-5)$	$(2x+3)(2x+3)$	$(2a-5)^2$
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In the questions above the polynomial product is three terms ( $ax^2 + bx + c$ ). What do you notice about the middle term ( $bx$ )?

These Trinomials are called \_\_\_\_\_ trinomials.

If you recognize that a polynomial is a perfect square trinomial, then you can factor it a certain way.

$$x^2 + 6x + 9$$

**Note:** Perfect square trinomials can still be factored the same way as any other trinomial.

Factor each of the following:

$a^2 + 12a + 36$	$100 - 20w + w^2$	$3x^2 + 30x + 75$	$4x^2 + 20x + 25$
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### ***More factoring.....***

Sometimes you might run into these situations. Factor:

$x^4 - 81$	$(a-5)^2 - 36$	$y^4 - 10y^2 + 25$
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## 5.4 Extra Practice

1. Determine each product.
  - a)  $(x + 14)(x - 14)$
  - b)  $(2a - 7)(2a + 7)$
  - c)  $(11x + 1)(11x - 1)$
  - d)  $(5y - 9)(5y + 9)$
  - e)  $(x^2 + 3)(x^2 - 3)$
2. What is each product?
  - a)  $(y + 10)^2$
  - b)  $(8 - m)^2$
  - c)  $(2a - 5k)^2$
  - d)  $4(3x - y)^2$
  - e)  $(x^2 + 5)^2$
3. Determine the missing values.
  - a)  $4p^2 - 25 = (2p)^2 - (\square)^2$
  - b)  $16x^2 - 9 = (\square)^2 - (\square)^2$
  - c)  $y^2 - 144 = (y - \square)(y + \square)$
  - d)  $9n^2 - 1 = (3n + \square)(3n - \square)$
  - e)  $x^4 - 49 = (x^2 - 7)(\square)$
4. What are the missing values?
  - a)  $x^2 + 10x + 25 = (x + \square)^2$
  - b)  $225 + 30p + p^2 = (\square + p)^2$
  - c)  $y^2 - 8y + \square = (y - 4)^2$
  - d)  $x^2 + \square + 121 = (x - 11)^2$
  - e)  $\square - 20w + w^2 = (10 - w)^2$
5. Factor each binomial, if possible.
  - a)  $x^2 - 144$
  - b)  $a^2 - 9b^2$
  - c)  $25x^2 - y^2$
  - d)  $h^2 + 64$
  - e)  $36 - a^2b^2$
6. Factor each trinomial, if possible.
  - a)  $x^2 + 14x + 49$
  - b)  $y^2 - 40y + 400$
  - c)  $36 + 12a + a^2$
  - d)  $64a^2 - 48ab + 9b^2$
  - e)  $16x^2 - 56xy + 49y^2$
7. Factor completely.
  - a)  $16x^2 - 4y^2$
  - b)  $9x^3 - 36x$
  - c)  $27a^4 - 147$
  - d)  $100ab^2 - 25a$
  - e)  $x^4 - 81$
8. Factor completely.
  - a)  $y^4 - 10y^2 + 25$
  - b)  $x^4 - 2x^2 + 1$
  - c)  $100a^2 - 100ab + 25b^2$
  - d)  $2x^3 + 40x^2y + 200xy^2$
  - e)  $y^4 + 18y^2 + 81$
9. Factor completely.
  - a)  $(x + 4)^2 - 25$
  - b)  $(a - 5)^2 - 36$
  - c)  $100 - (p + 8)^2$
  - d)  $(x + 2)^2 - (x - 2)^2$
  - e)  $x^2 - (y + z)^2$
10. Identify two values of  $n$  so that each polynomial will be a perfect square trinomial. Then, factor.
  - a)  $x^2 + nx + 64$
  - b)  $y + ny + 144$
  - c)  $4a^2 + na + 25$
  - d)  $9x^2 + nxy + 16y^2$
  - e)  $25x^2 + nx + 121$

**Section 5.4 Extra Practice**

1. a)  $x^2 - 196$  b)  $4a^2 - 49$  c)  $121x^2 - 1$   
d)  $25y^2 - 81$  e)  $x^4 - 9$   
2. a)  $y^2 + 20y + 100$  b)  $64 - 16m + m^2$   
c)  $4a^2 - 20ak + 25k^2$  d)  $36x^2 - 24xy + 4y^2$   
e)  $x^4 + 10x^2 + 25$   
3. a)  $\square = 5$  b)  $(4x)^2 - (3)^2$  c)  $(y - 12)(y + 12)$   
d)  $(3n + 1)(3n - 1)$  e)  $\square = x^2 + 7$   
4. a)  $\square = 5$  b)  $\square = 15$  c)  $\square = 16$   
d)  $\square = -22x$  e)  $\square = 100$   
5. a)  $(x - 12)(x + 12)$  b)  $(a - 3b)(a + 3b)$   
c)  $(5x - y)(5x + y)$  d) not possible  
e)  $(6 - ab)(6 + ab)$

6. a)  $(x + 7)^2$  b)  $(y - 20)^2$  c)  $(6 + a)^2$   
d)  $(8a - 3b)^2$  e)  $(4x - 7y)^2$   
7. a)  $4(2x - y)(2x + y)$  b)  $9x(x - 2)(x + 2)$   
c)  $3(3a^2 - 7)(3a^2 + 7)$  d)  $25a(2b - 1)(2b + 1)$   
e)  $(x - 3)(x + 3)(x^2 + 9)$   
8. a)  $(y^2 - 5)^2$  b)  $(x - 1)^2(x + 1)^2$  c)  $25(2a - b)^2$   
d)  $2x(x + 10y)^2$  e)  $(y^2 + 9)^2$   
9. a)  $(x - 1)(x + 9)$  b)  $(a - 11)(a + 1)$   
c)  $(2 - p)(18 + p)$  d)  $8x$  e)  $(x - y - z)(x + y + z)$   
10. a) Example:  $n = 16$ ;  $(x + 8)^2$  or  $n = -16$ ;  $(x - 8)^2$   
b) Example:  $n = 24$ ;  $(y + 12)^2$  or  $n = -24$ ;  $(y - 12)^2$   
c) Example:  $n = 20$ ;  $(2a + 5)^2$  or  $n = -20$ ;  $(2a - 5)^2$   
d) Example:  $n = 24$ ;  $(3x + 4y)^2$  or  $n = -24$ ;  $(3x - 4y)^2$   
e) Example:  $n = 110$ ;  $(5x + 11)^2$  or  $n = -110$ ;  $(5x - 11)^2$

