Expand + Simplify

multiply out factors — collect any like terms.

Factor

take answer and change it back into a multiplication.

Square 5.2: Common Factors

The opposite of the distributive property (when we expand products) is factoring. **Factoring** converts polynomials into simpler terms. This is often important for doing more complex mathematical operations.

When factoring a polynomial, always look for the Greatest Common Factor (GCF) first.

There are two ways to do this: Prime Factorization and Listing Factors

For example, determine the GCF of:

$$16x^2y \text{ and } 24x^2y^3$$

Prime Factorization	Listing Factors
$16x^2y$: 4, 4; 2, 2, 2, 2 $2\cdot2\cdot2\cdot2\cdot x\cdot x\cdot y$ $24x^2y^3$: 3, 8; 4, 2; 2, 2 $2\cdot2\cdot2\cdot3\cdot x\cdot x\cdot y\cdot y\cdot y$	16: 2, 8, 4, 1, 16 24: 1, 2, 3, 4, 6, 8, 12, 24 $8x^2y$

GCF = $2\cdot2\cdot2\cdot x\cdot x\cdot y = 8x^2y$

We can do this for polynomials as well:

Factor each of the following:

$10y-20$ GCF: 10 y, -2; 10: $10y$, -20 $\boxed{(10)(y-2)}$	$35a+10a^2$ GCF: $5a$ 7, $2a$; $5a$: $35a$, $10a^2$ $\boxed{5a(7+2a)}$	$24m^2n+16mn^2$ GCF: $8mn$ $8mn(3m+2n)$
$-28x^2y-35xy^2$ GCF: $-7xy$ $(-7xy)(4x+5y)$	$3x^2+12x-6$ GCF: 3 $3(x^2+4x-2)$	x^5-x^4 GCF: x^4 $x^4(x-1)$

Greatest common factors are not restricted to monomials. Sometimes a GCF can be a binomial or other polynomial.

Example: $9x(x-1)+2(x-1)$

GCF: $\underline{\underline{x-1}}$

$(x-1)(9x+2)$

Factor the following

$10x(x-3)+7(x-3)$ $(x-3)(10x+7)$	$3x^2(x-7)+2x(x-7)-4(x-7)$ $(x-7)(3x^2+2x-4)$
$3x(x^2+y^2)-5(x^2+y^2)$ $(x^2+y^2)(3x-5)$	$7a(a+2b)-(a+2b)$ $(a+2b)(7a-1)$ or $(7a-1)(a+2b)$

note: you <u>can</u> multiply in any order

Note that some binomial factors are opposites of each other; they have a GCF of <u>-1</u>

$(a+b)$, $(b+a)$ <u>same</u>

$(a-b)$ $(-b+a)$ <u>same</u>

$(a-b)$ $(b-a)$ not same, but opposites

Example: $(a-b) = (-1)(b-a)$

Factor the following

$2x(x-3)+3(3-x)$ $2x(x-3)+3(-1)(x-3)$ $2x(x-3)-3(x-3)$ $(x-3)(2x-3)$	$4x(x+1)-7(1+x)$ $(4x-7)(x+1)$

Sometimes you may need to break the polynomial into groups and factor each part separately:

Example: $\underbrace{5m^2 + 10mn}\underbrace{-3m - 6n}$

$5m(m+2n) -3(m+2n)$

$(m+2n)(5m-3)$

	m	$2n$
$5m$	$5m^2$	$10mn$
-3	$-3m$	$-6n$

Factor each of the following by grouping

$3p^2 - 6pq + 5p - 10q$	$2m^2 - 6mn \boxed{-3m + 9n}$
$3p(p-2q) + 5(p-2q)$ $(p-2q)(3p+5)$	$2m(m-3n) -3(m-3n)$ $(2m-3)(m-3n)$ $2m(m-3n) + 3(-m+3n)$ $2m(m-3n) + 3(-1)(m-3n)$ $(2m-3)(m-3n)$

Application

Paula has 18 toonies, 30 loonies, and 48 quarters. She wants to group her money so that each group has the same number of each coin and there are no coins left over.

a) What is the maximum # of groups she can make (Hint: Identify the GCF).

$18t + 30l + 48q$

GCF = 6

b) How many of each coin will be in each group?

$6(3t + 5l + 8q)$

c) How much money will each group be worth?

$3(2) + 5(1) + 8(.25)$

= \$13 for each group.

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