Warmup 2.9

1. Determine the derivative of the following functions

a) $y = x^3 f(x)$ b) $y = x^3 + f(x)$ c) $y = \dfrac{x^3}{f(x)}$

$3x^2 \cdot f(x) + f'(x)(x^3)$ $y' = 3x^2 + f'(x)$ $y' = \dfrac{3x^2 \cdot f(x) - f'(x) \cdot x^3}{f(x)^2}$

2. Use your calculator to find the derivative of $y = |16 - x^4|$ at $x = 2$. Is this a reasonable value?

nDeriv (abs(16-x^4), x, 2) = .024

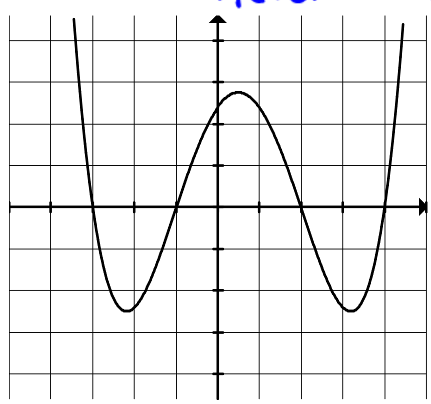
actually not differentiable at x=2 ; corner.

3. If $f(x) = \dfrac{x^3 - 3x}{(x^2+1)^3}$, then determine $f'(2.356)$ using your calculator. Is this a reasonable value?

nDeriv ((x^3-3x) / (x^2+1)^3, x, 2.356) = .0024

reasonable, it is differentiable at 2.356

4. To the right is the graph of $y = f'(x)$. For what values of x does the graph of $y = f(x)$ have a horizontal tangent?



f(x) has 4 horizontal tangents

when f'(x)=0

x = -3, -1, 2, 4

5. The height of a rock thrown into the air is given by the equation $h = 40t - 5t^2$. What is the maximum height reached by the rock? What is its velocity when it hits the ground?

The Chain Rule

A rate is a comparison of two things in different units

A certain machine costs $200/hr to rent. A company plans to rent the machine for 8 hours per day. What would the daily rate be to rent the machine?

$$\$200/\cancel{hr} \times 8\,\cancel{hr}/1\,day = \$1600/day.$$

A certain medication is to be administered at a dosage of 6 mg/kg. A sample of 20 students, each weighing 80 kg is to be administered the medication. How much medication is required to give the appropriate dose to a sample of this size?

$$\frac{6mg}{\cancel{kg}} \times \frac{80\,\cancel{kg}}{1\,\cancel{student}} \times \frac{20\,\cancel{students}}{1\,sample} = \frac{9600\ mg}{sample.}$$

y changes 6 times the rate of *u*. *u* changes 3 times the rate of *x*. How does the rate at which *y* changes compare with the rate at which *x* changes?

$$\frac{dy}{du} = 6 \qquad \frac{du}{dx} = 3 \qquad \frac{dy}{dx} = \frac{dy}{\cancel{du}} \cdot \frac{\cancel{du}}{dx}$$

$$= 6 \times 3 = 18$$

The above suggests

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

or

$$\frac{d}{dx} f(v) = \frac{df}{dv} \cdot \frac{dv}{dx}$$

or

$$\frac{d}{dx} f(g(x)) = \frac{df(g(x))}{dg(x)} \cdot \frac{dg(x)}{dx} = \boxed{f'(g(x)) \cdot g'(x)}$$

The chain rule is one of the most important and often used rules in differential calculus. Make sure that you understand what it says and how to use it correctly. In practice, the chain rule says to differentiate from the outside in, or from the outermost to the innermost function.

1. Determine the following derivatives

$\frac{d}{dx}x^3$ $\frac{d}{dv}v^3$ $\frac{d}{d(x+1)}(x+1)^3$ $\frac{d(3x^2-7x+4)^3}{d(3x^2-7x+4)}$

$3x^2$ $3v^2$ $3(x+1)^2$ $3(3x^2-7x+4)^2$

2. Determine the derivative of $y=(4x^2+7)^3$

First using the product rule:

$$y=(4x^2+7)\underbrace{(4x^2+7)(4x^2+7)}_{(4x^2+7)^2}$$

$$u=(4x^2+7)(4x^2+7)$$

$$\frac{du}{dx}=8x(4x^2+7)+8x(4x^2+7)$$

$$=16x(4x^2+7)$$

$$y'=(8x)(4x^2+7)^2+16x(4x^2+7)\cdot(4x^2+7)$$

Now using the chain rule

3. Determine the derivative: $\frac{d}{dx}(f(x))^n$

Determine the derivatives of the following:

1. $y=\left(3x^2-5\right)^4$	2. $y=\sqrt{8-5u}$
3. $y=\sin 3t$	4. $y=\cos^3 x$
5. $y=-18\sin\left(\frac{5}{x}\right)$	6. $y=\sqrt{\tan\theta}$
7. $y=\sin\left(\tan x\right)$	8. $y=f\left(g\left(h\left(x\right)\right)\right)$
9. $g\left(u\right)=u^{\frac{3}{5}}$, $u\left(x\right)=x^3-6x$. Determine $\frac{dg}{dx}$	10. Determine $\frac{df\left(t\right)}{dx}$

The Chain Rule

Determine the derivative of the following functions

1. $y = \sin^3 x \ \tan x$

2. $y = f(x^3)$

3. $y = \sqrt{(x^3 + 3)^2 + 4x^2}$

4. $y = \sin(\cos x)$

5. $y = \sqrt{\cos(8x)}$

6. $y = \dfrac{-5}{\sqrt{2 + x^2}}$

7. Let $y = x^2 + 2x - 3$. Determine $\frac{dy}{dt}$ when $x = 3$ and $\frac{dx}{dt} = -4$

8. Suppose that the functions f and g and their derivatives have the following values at $x = 1$ and $x = 2$

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	–4	9	–2	–5
2	3	1	5	3

Determine the derivatives of the following at the specified values:

a) $5f(x)$ at $x = 2$

b) $f(g(x))$ at $x = 2$

c) $\sqrt{g(x)}$ at $x = 1$

d) $\dfrac{1}{f^2(x)}$ at $x = 2$

e) $\sqrt{f^2(x) + g^2(x)}$ at $x = 1$

9. The height, h metres, of someone on a Ferris wheel is given by the equation $h = 18\cos\frac{2\pi}{50}(t - 3) + 17$ where t is the time in seconds.

a) At what time is the height increasing the fastest?

b) About how many meters per second is the height increasing when it is increasing at its fastest?

2.11 Warmup

Determine the derivative of the following functions

1. $y = \cos^4\left(\sqrt{2x^3+1}\right)$

2. $y = f\left(g\left(5x\right)\right)$

3. $y = \sqrt{f^3\left(x\right) + 9x^2}$

4. $y = \dfrac{-18}{\sin^2\left(2x-1\right)}$

5. $y = \dfrac{\cos\left(8x\right)}{\sin\left(8x\right)}$

6. Suppose that the functions f and g and their derivatives have the following values at $x = 1$ and $x = 2$

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	–4	9	–2	–5
2	3	1	5	3

a) $\frac{d}{dx}\frac{-5}{f^3(x)}$ at $x=2$

b) $\frac{d}{dx}\sqrt{f^3(x) + g(x^3)}$ at $x=1$