Applications

1. A bacteria culture has a population, n, after t hours given by $n = 400 + 280t + 49t^2$

a) Find $\frac{dn}{dt}$ and interpret the result. - how quickly is the number/amount changing

$$\frac{dn}{dt} = 98t + 280$$

b) Find the rate of growth after 3 hours.

$$98(3) + 280 = 574 \ \frac{\text{bacteria}}{\text{hour}}$$

2. In manufacturing, the cost of production *c(x)* is a function of *x*, the number of units produced. The **marginal cost of production** is the rate of change of cost with respect to the level of production and can thus be denoted as $\frac{dc}{dx}$. This is sometimes loosely defined to be the extra cost of producing 1 more unit, and is acceptable if the slope of *c* is not quickly changing near *x*.

The cost in dollars of producing *x* DVD drives is $c(x) = (50 - .1x)(40 + x)$.

a) What is the average cost of producing 100 DVD drives ?

$c(100)$ = cost of 100 drives

$$\text{average cost} = \frac{c(100)}{100} = \frac{(50-10)(40+100)}{100} = \$56/\text{drive}$$

b) What is the marginal cost of producing the 100th DVD drive ?

$$c(x) = (50 - .1x)(40 + x)$$

$$c'(x) = (-.1)(40+x) + (50-.1x)(1)$$

$$= -4 - .1x + 50 - .1x = 46 - 0.2x$$

c) What is the cost of producing the 100th DVD drive ?

$$\text{marginal cost} = c'(100) = 46 - 0.2(100)$$

$$= \$26$$

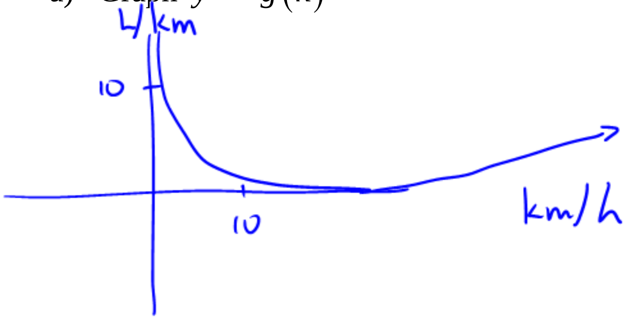
$$c(100) - c(99)$$

$$= 5600 - 5573.9$$

$$= \$26.10$$

3. A car burns gas at a rate of $g(x) = \left(\dfrac{1000 + x^2}{200x}\right)$ liters per kilometer when traveling at x km/h

a) Graph $y = g(x)$



b) What is $g'(x)$ and how do you interpret it ?

how does L/km change as speed increases

$$\frac{(2x)(200x) - (1000 + x^2)(200)}{(200x)^2} = \frac{400x^2 - 200000 - 200x^2}{(200x)^2}$$

$$= \frac{200x^2 - 200000}{40000x^2}$$

$$= \frac{x^2 - 1000}{200x^2} \quad \frac{\text{L/km}}{\text{km/h}}$$

c) Determine i) $g'(60)$ ii) $g'(80)$ iii) $g'(110)$

$$g'(60) = \frac{60^2 - 1000}{200(60)^2}$$

$$= 0.0036111 \ \frac{\text{L/km}}{\text{km/h}}$$

$$g'(80) = .0042 \ \frac{\text{L/km}}{\text{km/h}}$$

$$g'(110) = .0046 \ \frac{\text{L/km}}{\text{km/h}}$$

d) What is the most efficient speed for fuel consumption ?

→ fuel consumption is a minimum at the lowest point on graph when rate has a sign change.

$$g'(x) = 0$$

$$\frac{x^2 - 1000}{200x^2} = 0$$

$$x^2 - 1000 = 0$$

$$x^2 = 1000$$

$$x = \pm\sqrt{1000} = \pm 31.6 \text{ km/h}$$

$$\boxed{31.6 \text{ km/h}}$$

HW: p130