***2.6 Warmup***

1. Determine $\frac{d}{dx}\left(\frac{fg}{h}\right)$

2. Determine $\frac{d}{dx}f^3$ (Hint: Treat $f^3$ as $f \bullet f \bullet f$ and use the product rule)

3. Determine $\frac{d}{dt}\left(\frac{At-1}{A^2t}\right)$ where $A$ is a constant.

4. If $y = x^n$ what is $y^{(n)}$ or $\frac{d^n y}{dx^n}$ (or what is the $n^{th}$ derivative)?

5. If you keep on differentiating a polynomial function, will the derivative eventually become zero? Why or why not?

# *Velocity and Acceleration*

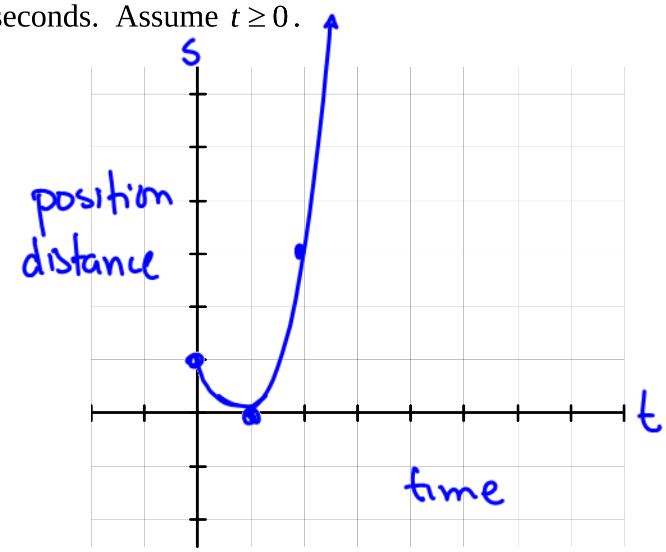
For any function $s = f(t)$ where $s$ = distance (or position or displacement) and $t$ = time, $\frac{\Delta s}{\Delta t}$ is the average velocity, $s'$ is the instantaneous velocity $v(t)$, and $s''$ is the instantaneous acceleration. Note that the average aceleration is $\frac{\Delta v}{\Delta t}$ and that $s''(t) = \frac{d^2 s}{dt^2} = v'(t) = \frac{dv}{dt} = a(t)$. Also speed is defined to be $|v(t)|$

Part One: **Straight Line Motion**

**Example**: A particle moves along a horizontal line according to $s = t^3 - t^2 - t + 1$ where $s$ is the distance in centimeters and $t$ is the time in seconds. Assume $t \geq 0$.

a) Sketch the graph of $s = f(t)$

| t | s |
|---|---|
| 0 | 1 |
| 1 | 0 |
| 2 | 3 |
| 3 | 16 |




b) Find the average velocity during the first 3 seconds.

$$\text{secant} = \frac{s(3) - s(0)}{3 - 0} = \frac{16 - 1}{3}$$

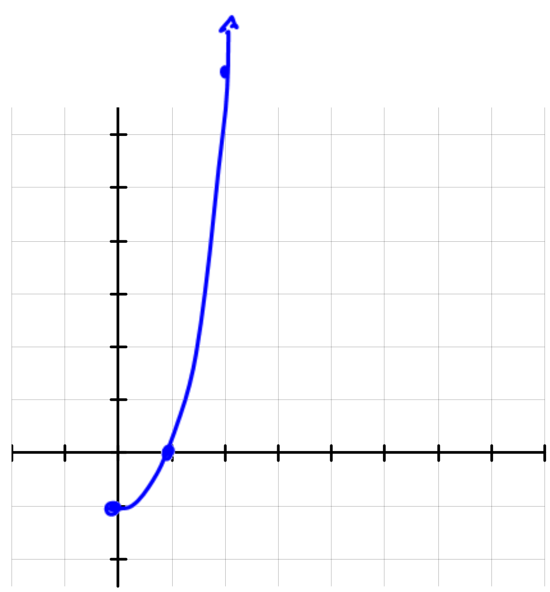
average $\bar{v} = 5$ m/s

c) Find the instantaneous velocity at time $t$ and sketch the graph of the velocity function.

$v(t) = s'(t)$ 1st derivative.

$v(t) = s'(t) = 3t^2 - 2t - 1$

| t | s'(t) |
|---|---|
| 0 | -1 |
| 1 | 0 |
| 2 | 7 |



d) Find the velocity at $t = 2$ s

$v(t) = s'(2) = 3(2)^2 - 2(2) - 1$

$= s'(2) = 7$

e) When is the velocity zero?

$0 = 3t^2 - 2t - 1$

$= 3t^2 - 3t + t - 1$

$= 3t(t-1) + 1(t-1)$

$0 = (3t+1)(t-1)$

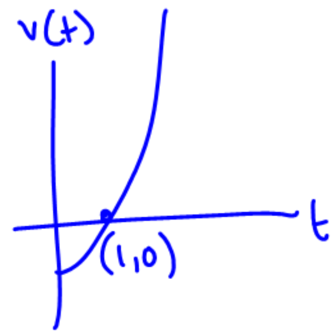
$\cancel{t = -\frac{1}{3}}$ or $t = 1$

since $t \geq 0$

f) When is the particle moving to the left?

when $v(t)$ is negative.

$v(t) < 0$ when $0 \leq t < 1$




g) What is the acceleration at time $t$? At $t = 2$ s?

$a(t) = v'(t) = s''(t)$

$v(t) = 3t^2 - 2t - 1$

$a(t) = 6t - 2$

$a(2) = 6(2) - 2$
$= 10$

h) What is the acceleration when the velocity is a minimum?

when mtan of velocity graph = 0

there will be a local maximum or minimum any time the derivative = 0

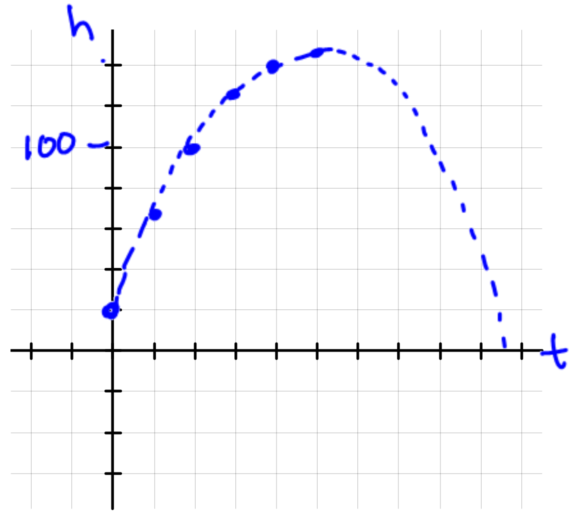
$a = 0$

Part Two: **Projectile Motion**

**Example**: A projectile is fired into the air from the top of a 20 m high hill. Its initial velocity is 50 m/s and its height after $t$ seconds is given by the function $h = -5t^2 + 50t + 20$.

a) Sketch the graph of $h = f(t)$

| t | h |
|---|---|
| 0 | 20 |
| 1 | 65 |
| 2 | 100 |
| 3 | 125 |
| 4 | 140 |
| 5 | 145 |




b) Find the velocity at time $t$. At $t = 2$ s.

$v(t) = h'(t) = -10t + 50$

$v(2) = -10(2) + 50$

$v(2) = 30$

c) How long will it take to reach its maximum height?

when does $h'(t) = 0$

$-10t + 50 = 0$

$10t = 50$

$\underline{\underline{t = 5\text{ s}}}$

d) What is the maximum height reached?

$h(5) = -5(5)^2 + 50(5) + 20$

$= 145$ m

e) What is the acceleration at time *t* ?

$a(t) = h''(t)$ or $a(t) = v'(t)$

$v(t) = -10t + 50$

$a(t) = v'(t) = -10$

f) What is the projectile's velocity when it hits the ground?

$h(t) = 0$ when it hits the ground.

need to find $t$ when $h = 0$

$0 = -5t^2 + 50t + 20$

$0 = -5(t^2 - 10t - 4)$

$t = \dfrac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(-4)}}{2(1)}$

$= \dfrac{10 \pm \sqrt{116}}{2}$

$t = 10.4$ or $\cancel{-.4}$

$v(t) = -10t + 50$

$v(10.4) = -10(10.4) + 50$

$\boxed{= -54}$