2.1 Warmup

Find the instantaneous rate of change of the volume of a sphere with respect to the radius when the radius is 10 cm. ($V = \frac{4}{3}\pi r^3$). Alternately, how quickly is the volume changing when the radius is 10 cm. ?

$$m_{tan} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{4}{3}\pi (a+h)^3 - \frac{4}{3}\pi (a)^3}{h}$$

$$= \lim_{h \to 0} \frac{\frac{4}{3}\pi \left[(a+h)^3 - a^3\right]}{h}$$

$$= \lim_{h \to 0} \frac{\frac{4}{3}\pi \left(\cancel{a^3} + 3a^2h + 3ah^2 + h^3 - \cancel{a^3}\right)}{h}$$

$$= \lim_{h \to 0} \frac{4}{3}\pi \left(3a^2 + 3ah + h^2\right)$$

$$m_{tan} = 4\pi a^2$$

$a = 10$

$$m_{tan} = 4\pi (10)^2$$

$$= 400\pi$$

Definition of the Derivative

a derivative is a function to determine instantaneous rate of change.

The ***derivative*** of the function $y = f(x)$ with respect to x is the function

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \qquad \text{or} \qquad f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

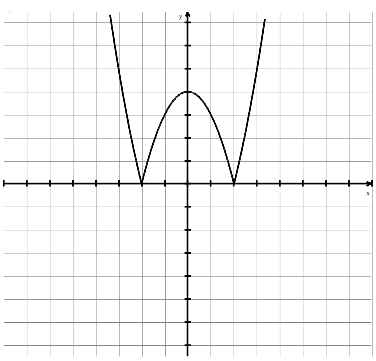
$f(x)$ = function

$f'(x)$ = tangent to the function

The derivative is a **function** that tells you

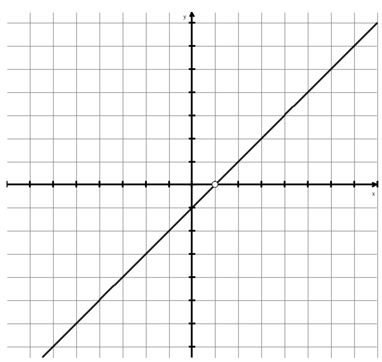
1. slope of curve at any point on the function
2. slope of the tangent line
3. instantaneous rate of change.

The domain of f' is not necessarily the same as the domain of f (alternately, the tangent does not have to exist at each point where the function is defined)



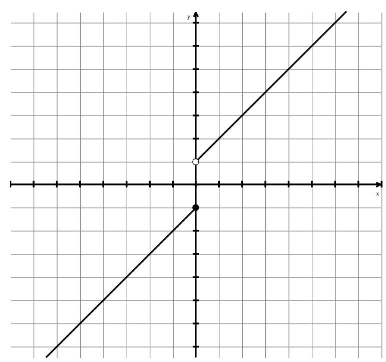
$f(x)$: D : $x \in \mathbb{R}$

$f'(x)$: D : $x \neq \pm 2$



D: $x \neq 1$

$x \neq 1$



D: $x \in \mathbb{R}$

$x \neq 0$

If $f'(x)$ exists, we say that the function is **differentiable** at x. The process of finding the derivative of a function is called **differentiation**.

differentiable = you can find the tangent at that specific point.

Example If $\underbrace{f(3)=8}$ and $\underbrace{f'(3)=4}$, find the equation of the tangent to $y=f(x)$ at $x=3$.

↳ needs point and slope.

$(3,8)$ Slope $=4$ at $x=3$

$$\boxed{y-8=4(x-3)}$$

Notations for the derivative

Notation	Read as
y'	*y* prime – nice and compact
$f'(x)$	*f* prime of *x* – emphasizes that the derivative is a function and is related to $f(x)$
$\frac{dy}{dx}$	dee *y* dee *x* – shows that you are differentiating *y* with respect to *x*
$\frac{df}{dx}$	dee *f* dee *x* – shows that you are differentiating the function *f* with respect to *x*
$\frac{d}{dx}f(x)$	dee dee *x* of $f(x)$ - shows that differentiation is an operation being performed on the function $f(x)$

Example: Find the derivative of $y=x^2-5x$ **or** Find y' if $y=x^2-5x$ **or** Find $\frac{d}{dx}(x^2-5x)$

$$y' = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{[(x+h)^2-5(x+h)]-[x^2-5x]}{h}$$

$$= \lim_{h\to 0} \frac{[\cancel{x^2}+2xh+h^2-\cancel{5x}-5h]-[\cancel{x^2}-\cancel{5x}]}{h}$$

$$= \lim_{h\to 0} \frac{2xh+h^2-5h}{h}$$

$$y' = 2x-5$$

$x+h=a$

Alternate Definition for the Derivative at a Point

$$f'(a)=\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$$

instead of $h \to 0$ $f(x+h)$

$x \to a$ $f(a)$

Example: Use the alternate definition to differentiate $y=3\sqrt{x}$

$$y'=\lim_{x\to a}\frac{f(x)-f(a)}{x-a}=\lim_{x\to a}\frac{3\sqrt{x}-3\sqrt{a}}{x-a}\cdot\frac{3\sqrt{x}+3\sqrt{a}}{3\sqrt{x}+3\sqrt{a}}$$

$$=\lim_{x\to a}\frac{9x-9a}{(x-a)(3\sqrt{x}+3\sqrt{a})}$$

$$=\lim_{x\to a}\frac{9\cancel{(x-a)}}{\cancel{(x-a)}(3\sqrt{x}+3\sqrt{a})}$$

$$=\frac{9}{3\sqrt{a}+3\sqrt{a}}$$

$$=\frac{9}{6\sqrt{a}} \text{ or } \frac{3}{2\sqrt{a}}$$

What is the difference between $f'(x)$ and $\frac{df}{dx}$?

no difference

What is the difference between $f'(x)$ and $f'(a)$?

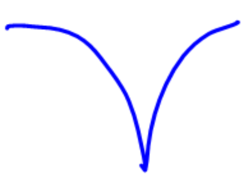
no difference $f'(a)$ what is $f'(x)$ at $x=a$

What is the difference between $f'(5)$ and $f'(x)$?

$f'(x)$ is the function to find m_{tan}

$f'(5)$ is m_{tan} specifically at $x=5$

Note In order for the derivative of a function to exist at a point (or for the function to be differentiable at that point), **one** of the requirements that must be met is that the left and right hand derivatives (instantaneous slopes) must be equivalent



not differentiable

the tangents on the left and right are not the same.

Express the following as limits. Do not determine the limit.

1. Find y' if $y = x^3 - 2x^2 + 8x$

$$y' = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} \quad \text{or} \quad y' = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{x \to a} \frac{(x^3 - 2x^2 + 8x) - (a^3 - 2a^2 + 8a)}{x - a}$$

$$\lim_{h \to 0} \frac{\left[(x+h)^3 - 2(x+h)^2 + 8(x+h)\right] - \left[x^3 - 2x^2 + 8x\right]}{h}$$

2. Find $f'(2)$ if $f(x) = x^3 - 2x^2 + 8x$

$$\lim_{x \to 2} \frac{(x^3 - 2x^2 + 8x) - (8 - 8 + 16)}{x - 2} \quad \text{or} \quad \lim_{h \to 0} \frac{\left((2+h)^3 - 2(2+h)^2 + 8(2+h)\right) - (8 - 8 + 16)}{h}$$

3. Find $\frac{dy}{dx}$ if $y = x^5$

$$\lim_{x \to a} \frac{x^5 - a^5}{x - a} \qquad \lim_{h \to 0} \frac{(x+h)^5 - (x)^5}{h}$$

4. Find $\left.\frac{dy}{dx}\right|_{x=3}$ if $y = x^5$

notation: find the derivative when $x = 3$

$$y'(3) = \lim_{x \to 3} \frac{x^5 - 3^5}{x - 3} \quad \text{or} \quad y'(3) = \lim_{h \to 0} \frac{(3+h)^5 - 3^5}{h}$$

$\boxed{\text{p 101 \#1-6, 11, 15}}$