Review: Slopes of Tangents and Normals

1. Find the equation of the tangent to the curve $y = \frac{1}{x}$ at the point where $x = 2$.

coord $2, f(2)$

$(2, \frac{1}{2})$

$m = \frac{-1}{4}$

$m_{tan} = \lim_{h \to 0} \frac{\frac{1}{a+h} - \frac{1}{a}}{h}$

$= \lim_{h \to 0} \frac{1}{h}\left(\frac{(a) - (a+h)}{(a+h)(a)}\right)$

$= \lim_{h \to 0} \frac{1}{h}\left(\frac{-h}{(a+h)(a)}\right) = \boxed{\frac{-1}{a^2}}$

$m_{tan} = \frac{-1}{(2)^2} = \frac{-1}{4}$

$\boxed{y - \frac{1}{2} = \frac{-1}{4}(x-2)}$

2. Find all points on the curve $y = x^3$ which have a slope of 6.

$m_{tan} = \lim_{h \to 0} \frac{(a+h)^3 - a^3}{h}$

$= \lim_{h \to 0} \frac{(a^3 + 3a^2h + 3ah^2 + h^3) - a^3}{h}$

$= \lim_{h \to 0} \frac{3a^2h + 3ah^2 + h^3}{h} = \frac{h(3a^2 + 3ah + h^2)}{h}$

$m_{tan} = 3a^2$

$6 = 3a^2$

$a^2 = 2$

$a = \pm\sqrt{2}$

$(\sqrt{2}, f(\sqrt{2})) = (\sqrt{2}, (\sqrt{2})^3)$

$-\sqrt{2}, f(\sqrt{2}) = (-\sqrt{2}, (-\sqrt{2})^3)$

3. Find the equation of the normal to the curve $y = x^2$ at $x = -3$.

need a coordinate

$(-3, f(-3)) = (-3, 9)$

$m_{tan} = 2x$

$m_{tan} = -6$

$m_{norm} = \frac{1}{6}$

$y - 9 = \frac{1}{6}(x+3)$

4. Determine the equation(s) of all tangents to the curve $y = x^2 + 1$ which pass through the point $(2, -4)$.

$m_{tan} = \lim_{h \to 0} \frac{[(a+h)^2+1] - [a^2+1]}{h}$

$m_{tan} = 2a$

coordinates are at (a, a^2+1)

$\frac{a^2+1 - -4}{a - 2} = 2a$

$a^2 + 5 = 2a^2 - 4a$

$0 = a^2 - 4a - 5$

$0 = (a-5)(a+1)$

$a = 5$ or -1

$f(5) = 26$ $\quad f(-1) = 2$

$m_{tan} = 10$ $\quad m_{tan} = -2$

$y - 26 = 10(x-5)$

and

$y - 2 = -2(x+1)$

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