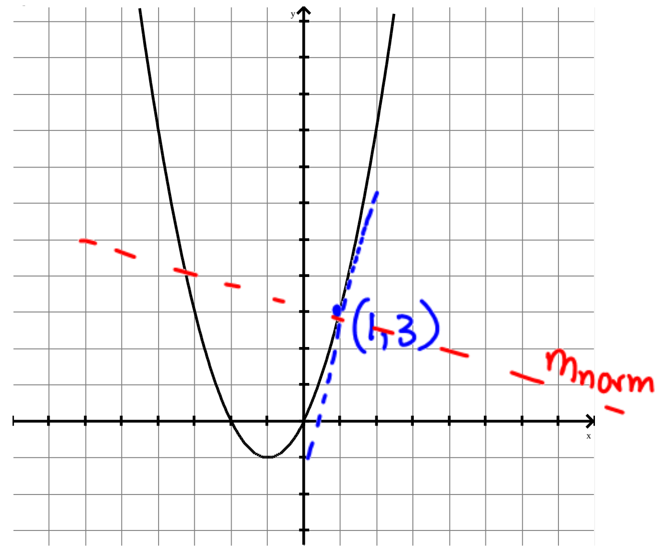
Tangents and Normals

1. For the function $y = x^2 + 2x$, determine



a) the slope of the curve at $x = 1$

- determine the tangent.

$$m_{tan} = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h \to 0} \frac{[h^2+2h+1 + 2(h+1)] - [3]}{h}$$

$$= \lim_{h \to 0} \frac{h^2+4h}{h} = \frac{\cancel{h}(h+4)}{\cancel{h}}$$

$m_{tan} = 4$

b) the instantaneous rate of change of the function at $x = 1$

4

c) the equation of the tangent to the curve at $x = 1$ (1,3)

use slope point form $y - 3 = 4(x-1)$

$y - y_1 = m(x - x_1)$

*normal line is perpendicular to the tangent line

d) the slope of the normal to the curve at $x = 1$

perpendicular slopes are negative reciprocals

$m_{tan} = 4$

$m_{norm} = \frac{-1}{4}$

e) the equation of the normal to the curve at $x = 1$

$y - 3 = \frac{-1}{4}(x-1)$

2. Find the slope of the tangent to the curve $y = f(x)$ at $x = a$

$$m_{tan} = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$

$$= \lim_{h\to 0} \frac{[(a+h)^2 + 2(a+h)] - [a^2+2a]}{h}$$

$$= \lim_{h\to 0} \frac{(\cancel{a^2}+2ah+h^2+\cancel{2a}+2h) - (\cancel{a^2}+\cancel{2a})}{h}$$

$$= \lim_{h\to 0} \frac{h(2a+h+2)}{h}$$

$m_{tan} = 2a+2$

3. At what point is the tangent to the curve $y = x^2 - 3x + 2$ horizontal ?

when is $m_{tan} = 0$?

$$m_{tan} = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$

$$= \lim_{h\to 0} \frac{[(a+h)^2 - 3(a+h) + 2] - [a^2 - 3a + 2]}{h}$$

$$= \lim_{h\to 0} \frac{(\cancel{a^2}+2ah+h^2-\cancel{3a}-3h+\cancel{2}) - (\cancel{a^2}-\cancel{3a}+\cancel{2})}{h}$$

$$= \lim_{h\to 0} \frac{h(2a+h-3)}{h}$$

$$= \lim_{h\to 0} 2a+h-3$$

$m_{tan} = 2a-3$

when does $2a-3=0$

$\boxed{a = \frac{3}{2}}$

$f(\frac{3}{2}) = (\frac{3}{2})^2 - 3(\frac{3}{2}) + 2$

$= \frac{9}{4} - \frac{9}{2} + 2$

$= \frac{9}{4} - \frac{18}{4} + \frac{8}{4}$

$f(\frac{3}{2}) = -\frac{1}{4}$

point at $= \left(\frac{3}{2}, -\frac{1}{4}\right)$

4. Find the equations of all tangents to the curve $y = \dfrac{1}{x+1}$ that have a slope of -1.

$$m_{tan} = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{1}{a+h+1} - \frac{1}{a+1}}{h}$$

$$= \lim_{h\to 0} \left(\frac{1}{h}\right)\left(\frac{(\cancel{a}+\cancel{1}) - (\cancel{a}+h+\cancel{1})}{(a+h+1)(a+1)}\right)$$

$$= \lim_{h\to 0} \left(\frac{1}{\cancel{h}}\right)\left(\frac{-\cancel{h}}{(a+h+1)(a+1)}\right)$$

$$m_{tan} = \frac{-1}{(a+1)^2}$$

$m_{tan} = \frac{-1}{(a+1)^2} = -1$

$\frac{(a+1)^2}{-1} = \frac{1}{-1}$

$(a+1)^2 = 1$

$a+1 = \pm\sqrt{1}$

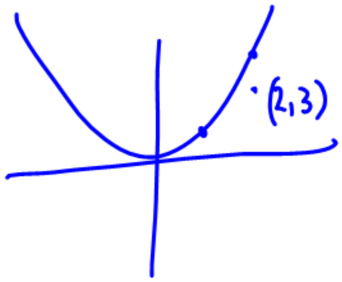
$a = -1 \pm 1$

$a = 0$ or -2

$f(0) = 1$ $\qquad$ $f(-2) = -1$

$\boxed{y-1 = -1(x-0)}$ $\qquad$ $\boxed{y+1 = -1(x+2)}$

5. Find the equations of all lines tangent to $y = x^2$ that pass through (2 , 3)



there are 2 tangent lines that pass through this point.

$$m_{tan} = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$

$$= \lim_{h\to 0} \frac{[(a+h)^2] - [a^2]}{h}$$

$$= \lim_{h\to 0} \frac{(\cancel{a^2} + 2ah + h^2) - (\cancel{a^2})}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(2a+h)}{\cancel{h}}$$

$$\boxed{m_{tan} = 2a}$$

slope between (a, a^2) and $(2,3)$ = m_{tan}

$$\frac{a^2-3}{a-2} = 2a$$

$$a^2 - 3 = 2a^2 - 4a$$

$$0 = a^2 - 4a + 3$$

$$0 = (a-3)(a-1)$$

$$a = 3 \text{ or } 1$$

$f(3) = 9$	$f(1) = 1$
$m_{tan} = 2a$	$m_{tan} = 2a$
$m_{tan} = 6$	$m_{tan} = 2$
$\boxed{y - 9 = 6(x-3)}$	$\boxed{y - 1 = 2(x-1)}$

p88 # 23-27 all
29, 31, 33, 35

Test on Friday