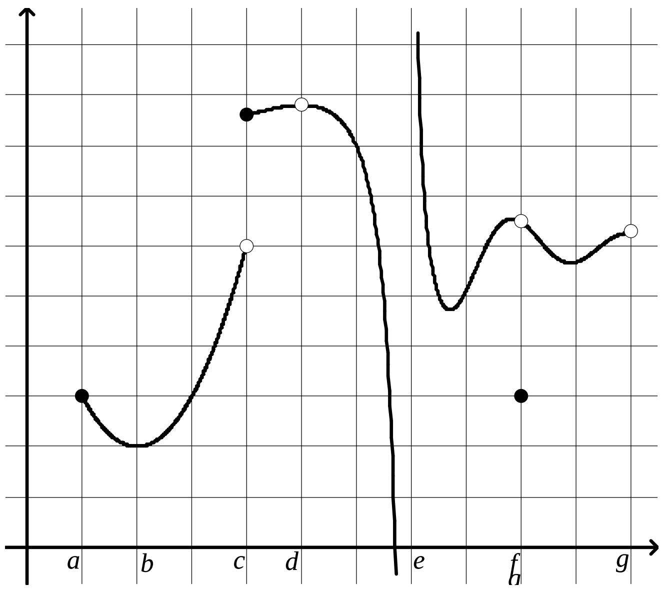
Continuity

Definition: A function is continuous at an interior point *c* of an interval [*a*,*b*], if and only if, $\lim_{x \to c} f(x) = f(c)$

Note: This definition defines continuity at an interior point of an interval.
In order for a function to be continuous at *a*, the limit must exist and the function has to be defined at *a*

This definition can be extended to define continuity at the left endpoint, *a*, of an interval:
$\lim_{x \to a^+} f(x) = f(a)$ or for the right endpoint, *b*,: $\lim_{x \to b^-} f(x) = f(b)$



a point can still be continuous if there is only a left or right hand limit

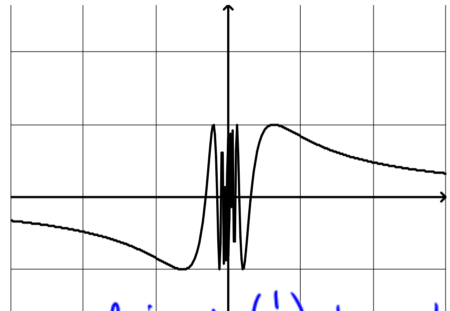
Point	*Continuous ?*	*Reason*	*Type of Discontinuity*
a	yes	$\lim_{x \to a^+} f(x) = f(a)$	
b	yes	$\lim_{x \to b} f(x) = f(b)$	
c	no	limit does not exist	jump discontinuity
d	no	f(x) does not exist for x=d	removable discontinuity
e	no	$\lim_{x \to e^-} f(x) = -\infty$, $\lim_{x \to e^+} f(x) = \infty$ f(e) does not exist	infinite
f	no	$\lim_{x \to f} f(x) \neq f(f)$	removable
g	no	$\lim_{x \to g} f(x) \neq f(g)$ f(g) does not exist	removable.

Another type of discontinuity is an ***oscillating discontinuity***, illustrated by the function $y = \sin\left(\frac{1}{x}\right)$

as $x \to 0$

$\frac{1}{x} \to$ very large, close to infinity.

$y = \sin\left(\frac{1}{x}\right)$ keeps oscillating between -1 and 1

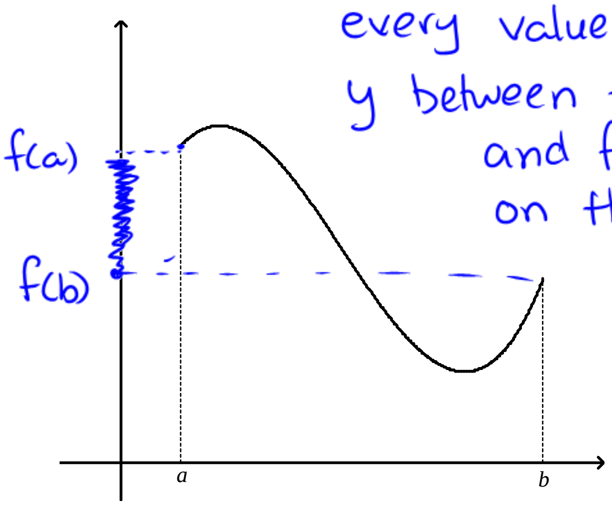


$\lim_{x \to 0} \sin\left(\frac{1}{x}\right)$ does not exist.

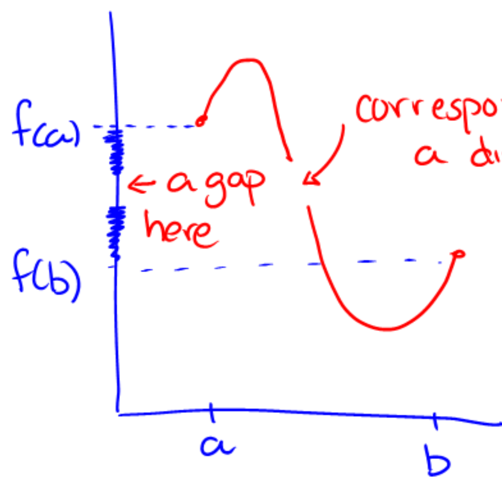
A function is said to be ***continuous on an interval*** if it is continuous at each point in the interval. A function is continuous if it is continuous at each point in its domain. Thus $y = \sqrt{x}$ and $y = \dfrac{1}{x}$ are both examples of continuous functions.

The Intermediate Value Theorem for Continuous Functions

If $f(x)$ is a continuous function on the interval $[a, b]$, then $f(x)$ takes on every value between $f(a)$ and $f(b)$



every value of y between f(b) and f(a) is on the graph.



corresponds to a discontinuity

Examples

1. Is any real number exactly 2 more than its cube? Explain.

let the number $= x$

$x^3 + 2 = x$

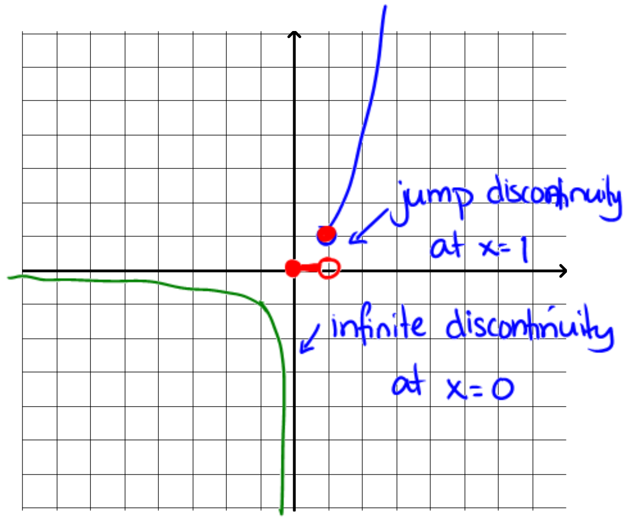
$x^3 - x + 2 = 0$ is there a solution

polynomial function is always continuous.

$\left.\begin{aligned} f(-2) &= -4 \\ f(0) &= 2 \end{aligned}\right\}$ $\therefore$ there is a solution where $f(x) = 0$, it occurs where $-2 < x < 0$

2. For the following function, determine any points of discontinuity and describe the type of discontinuity.

$$y = \begin{cases} \frac{1}{x} & x < 0 \\ \text{int}(x) & 0 \le x \le 1 \\ x^2 & x > 1 \end{cases}$$



3. Give a formula for the extended function that is continuous at the indicated point.

$$y = \frac{x-4}{\sqrt{x}-2} \cdot \frac{\sqrt{x}+2}{\sqrt{x}+2} \quad \text{at} \quad x = 4$$

find the limit

$$= \frac{\cancel{(x-4)}(\sqrt{x}+2)}{\cancel{x-4}}$$

$$y = \sqrt{x} + 2$$

$$\lim_{x \to 4} f(x) = 4$$

$\lim_{x \to 4} f(x) = f(4)$ is false

$\therefore$ not continuous because of the discontinuity at $x=4$

hole at $(4,4)$

$$y = \begin{cases} \frac{x-4}{\sqrt{x}-2} & x \neq 4 \\ 4 & x = 4 \end{cases}$$

or

$$y = \sqrt{x} + 2$$

4. Determine the value of k so that the function is continuous

$$y = \begin{cases} kx^2 - 1 & x \le 2 \\ -3x + 7 & x > 2 \end{cases}$$

left hand limit must equal right hand limit

discontinuous

continuous if the left piece joins up with right piece.

$$k(2)^2 - 1 = -3(2) + 7$$

$$4k - 1 = 1$$

$$4k = 2$$

$$\boxed{k = \tfrac{1}{2}}$$

p180 #1-9 odd
#11-18 all
#19-29 odd
#35-38, 41, 42, 45