Mean Value Theorem

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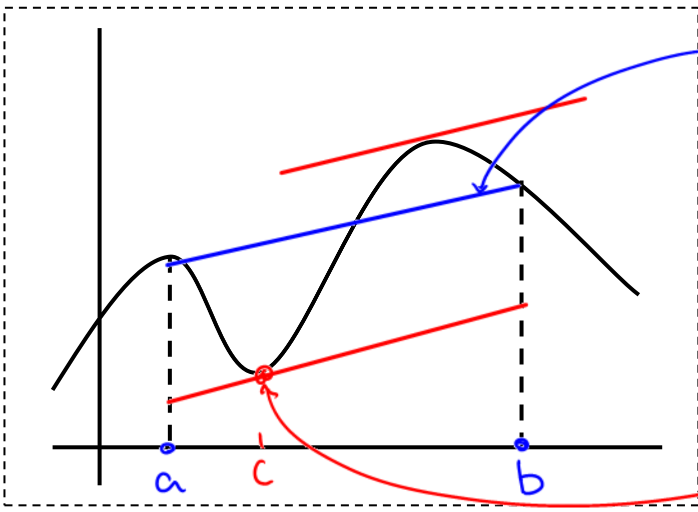
If f is continuous on $[a,b]$ and differentiable on (a,b), then there exists at least one point c in (a,b) such that

$$f'(c)=\underbrace{\frac{f(b)-f(a)}{b-a}}$$

secant line

Geometricaly, the secant line through the two points $(a,\ f(a))$ and $(b,\ f(b))$ is parallel to the tangent line at at least one point on f in $[a,b]$



secant line $m_{sec}=\dfrac{f(b)-f(a)}{b-a}$

this function has 2 places with $m_{tan}=m_{sec}$

m_{tan} at "c" has the same slope as m_{sec}

In physical terms, the Mean Value Theorem states that the average change in the function over an interval must equal the instantaneous change at some interior point of the interval.

1. Find the value for c in the interval $[2,4]$ which satisfies the Mean Value Theorem for the function $f(x)=3x^2-x+1$

find m_{sec}

$$\frac{f(4)-f(2)}{4-2}=\frac{45-11}{2}$$

$$=17$$

$y'=6x-1$

$17=6x-1$

$\boxed{x=3}$

2. Find the point on $[-8, 8]$ for the function $y = x^{\frac{2}{3}}$ which satisfies the Mean Value Theorem.

$$m_{sec} = \frac{f(8) - f(-8)}{8 - (-8)}$$

$$= \frac{4 - 4}{16}$$

$$m_{sec} = 0$$

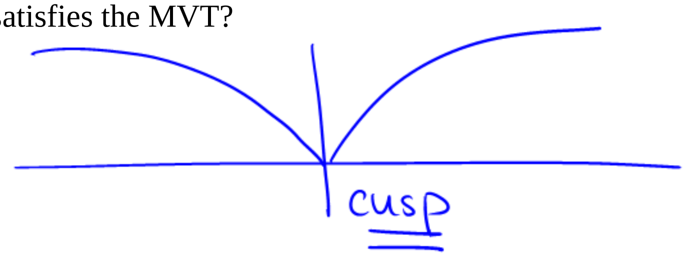
$$y' = \frac{2}{3}(x)^{-1/3}$$

$$= \frac{2}{3\sqrt[3]{x}}$$

this does not work because $y = x^{2/3}$ is not differentiable over the interval

Why is there no point in the interval which satisfies the MVT?

not differentiable at $x = 0$



Note: The hypotheses (requirements) of the MVT are f is continuous on $[a, b]$ **and** differentiable on (a, b). Functions which fail to meet these requirements may still possess the property stated by the MVT, but are not guaranteed to have it.

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