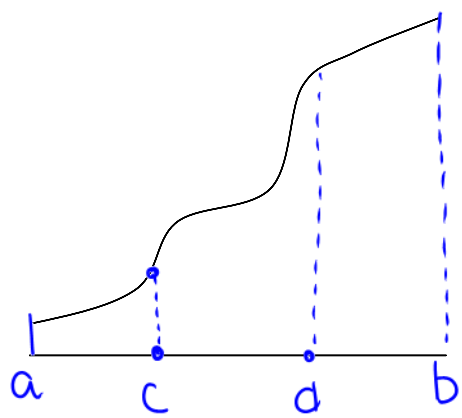
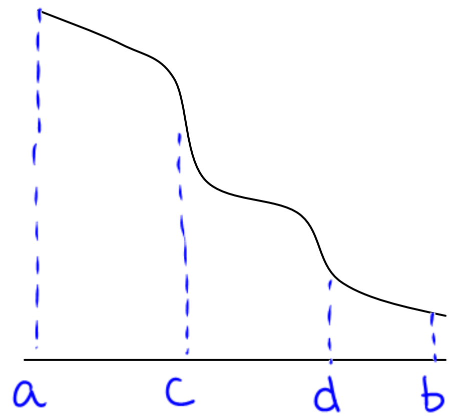
Increasing and Decreasing Functions

A continuous function is *increasing* / *decreasing* over $[a,b]$ if for all *c* and *d* with $a<c<d<b$, $f(c)<f(d)$ / $f(c)>f(d)$

Increasing Function



Decreasing Function



In terms of derivatives, we can say that if $f(x)$ is a function with continuous derivatives on an interval, then $f(x)$ is a(n) *increasing* / *decreasing* function on the interval if $f'(x)>0$ / $f'(x)<0$ for each *x* on the interval

In other words, the **sign** of the derivative tells you whether the function is increasing or decreasing.

In order for a continuous function to change from increasing to decreasing, it must past through a critical point.

If the derivative has any discontinuities, intervals bounded by any points of discontinuity for either *f* or f' are considered separately.

Find the intervals where the function is *i*) increasing *ii*) decreasing. Identify any extrema.

1. $y = x^4 + 2$

$y' = 4x^3$

$y' = 0$ when $x = 0$

x	-1	0	1
y'	-4	0	4
	↘		↗

decreasing on $(-\infty, 0)$

increasing on $(0, \infty)$

global min at $f(0) = 2$ when $x = 0$

2. $y = x^3 - 9x$

$y' = 3x^2 - 9$

crit points

$3x^2 - 9 = 0$

$3x^2 = 9$

$x^2 = 3$

$x = \pm\sqrt{3}$

	-2	$-\sqrt{3}$	0	$\sqrt{3}$	2
y'	3	0	-9	0	3
graph	$\nearrow$		$\searrow$		$\nearrow$

increasing on $(-\infty, -\sqrt{3})$ and $(\sqrt{3}, \infty)$

decreasing on $(-\sqrt{3}, \sqrt{3})$

local max $f(-\sqrt{3}) = 6\sqrt{3}$ when $x = -\sqrt{3}$

local min $f(\sqrt{3}) = -6\sqrt{3}$ when $x = \sqrt{3}$

3. $f'(x) = xe^x$ Note: This is the derivative of the function

crit points

$x = 0$

	-1	0	1
y'	$\frac{-1}{e}$	0	e
	$\searrow$		$\nearrow$

decreasing on $(-\infty, 0)$

increasing on $(0, \infty)$

global min occurs when $x = 0$, but we can't find $f(x)$ because we only know $f'(x)$