Critical Points

Find the extreme values of the following on the specified interval.

1. $f(x) = \dfrac{-1}{\sqrt{9-x^2}}$

what is the domain?

Den $\neq 0$ $\quad x = 3, -3$

$\sqrt{\text{neg}}$ $\quad x > 3,\ x < -3$

$\left.\right\}$ $-3 < x < 3$

$(-3, 3)$

$= -(9-x^2)^{-\frac{1}{2}}$

$f'(x) = +\frac{1}{2}(9-x^2)^{-\frac{3}{2}}(-2x) = \dfrac{-1x}{(9-x^2)^{3/2}}$

Critical points

$0 = \dfrac{-x}{(9-x^2)^{3/2}}$

$x = 0$

f' undefined at $x = \pm 3$, but these are not in dom.

x	$f(x)$
-2.9	"large" neg
0	$-1/3$
2.9	"large" neg

Global max of $-\frac{1}{3}$ at $x = 0$.

2. $y = x^3$ on $[-2, 3]$

$y' = 3x^2$

Critical points

$3x^2 = 0$

$x = 0$

y' undef? No

x	$f(x)$	
-2	-8	Global min of -8 at $x=-2$
0	0	Not an extreme
3	27	Global max of 27 @ $x=3$

3. $y = \sqrt[3]{x}$ on $[-2, 3]$

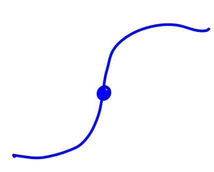
$y' = \frac{1}{3}x^{-2/3}$

Critical points

$0 = \dfrac{1}{3\sqrt[3]{x^2}}$

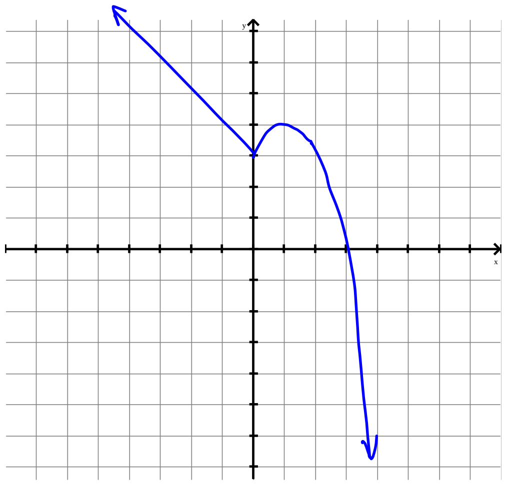
$x = \phi$

y' undef @ $x = 0$



x	$f(x)$	
-2	$\sqrt[3]{-2}$	Global min of $\sqrt[3]{-2}$ at $x=-2$
0	0	Not an extreme
3	$\sqrt[3]{3}$	Global max of $\sqrt[3]{3}$ at $x=3$

4. $y=\begin{cases}3-x & x<0\\ 3+2x-x^2 & x\geq 0\end{cases}$



Analysis without graph

Continuous at $x=0$ (left and right y values the same)

Left hand slope

$y'=-1$

Right hand slope

$y'=2-2x$

$y'(0)=2-0=2$

$\therefore$ y' does not exist at $x=0$.

Left of $x=0$

$y'=-1$

Right of $x=0$

$2-2x=0$

$-2x=-2$

$x=1$

(which is right of one)

x	$f(x)$
$-\infty$	∞
0	3
1	4
∞	$-\infty$

Local min of 3 @ $x=0$

Local max of 4 @ $x=1$

5. $f(x)=\ln\left|\dfrac{2x}{4+x^2}\right|$

★ Absolute value makes this hard to differentiate

$$= \begin{cases}\ln \dfrac{2x}{4+x^2} & x>0\\ \ln\left(-\dfrac{2x}{4+x^2}\right) & x<0\end{cases}$$

LHS

$y=\ln(-2x)-\ln(4+x^2)$

$y'=\dfrac{1}{-2x}\cdot -2-\dfrac{1}{4+x^2}\cdot 2x$

$0=\dfrac{1}{x}-\dfrac{2x}{4+x^2}$

$\dfrac{2x}{4+x^2}=\dfrac{1}{x}$

$2x^2=4+x^2$

$x^2=4$

$x=\pm 2$ want $x<0$

$x=-2$

RHS

$y=\ln 2x-\ln(4+x^2)$

$y'=\dfrac{1}{2x}\cdot 2-\dfrac{2x}{4+x^2}$

$0=\dfrac{1}{x}-\dfrac{2x}{4+x^2}$

$x=\pm 2$

want $x>0$

$x=2$

Must include something close to 0

x	$f(x)$
$-\infty$	$-\infty$
-2	$\ln(\frac{1}{2})$
0	$-\infty$
2	$\ln(\frac{1}{2})$
∞	$-\infty$

Global max of $\ln(\frac{1}{2})$ @ $x=\pm 2$