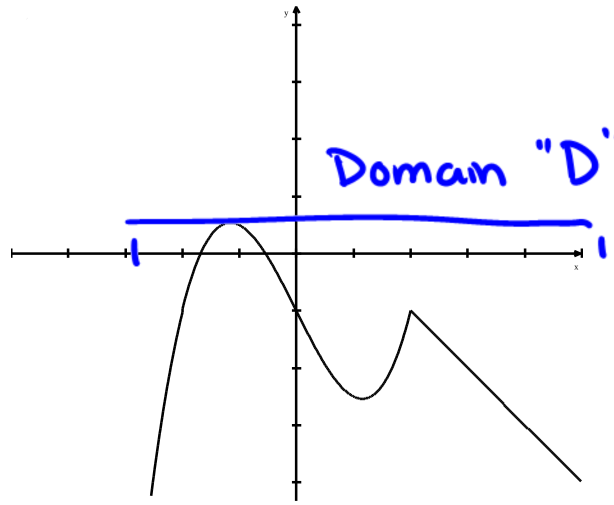
Extreme Values

Global (Absolute) Extreme Values

If $f(x)$ is a function defined on a domain D, then



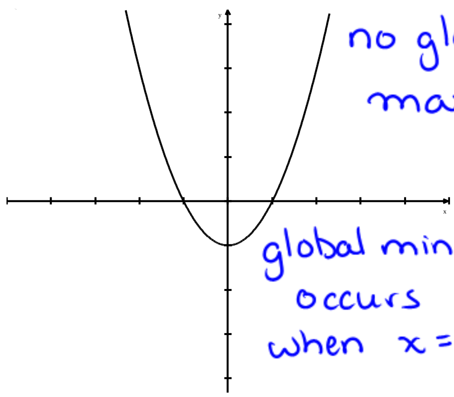
$f(c)$ is a global maximum on D iff.

$f(c) \geq f(x)$ where x is a value in D

$f(c)$ is a global minimum on D iff.

$f(c) \leq f(x)$ where x is a value in D

Graphed below is the function $y = x^2 - 1$ on differing domains as indicated. Determine any global extrema and where they occur.

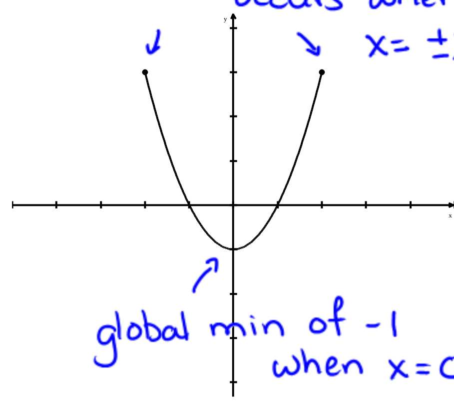


no global max.

global min of -1 occurs when $x=0$

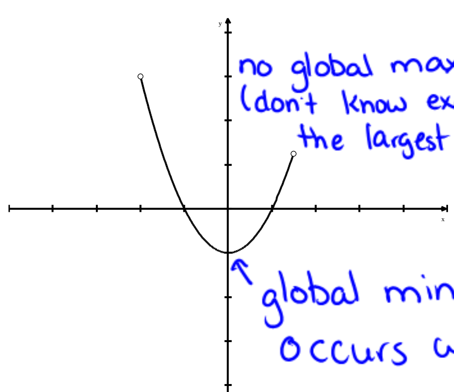
$(-\infty, \infty)$

global maximum of 3 occurs when $x = \pm 2$



global min of -1 when $x=0$

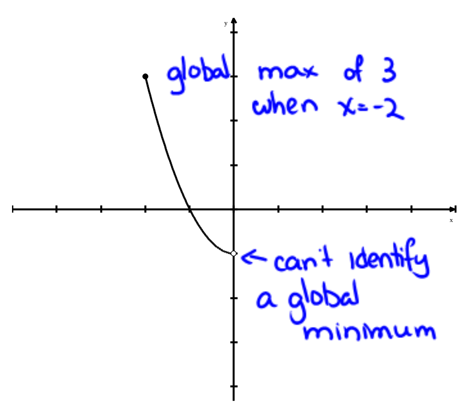
$[-2, 2]$



no global maximum (don't know exactly the largest value of f(x))

global min of -1 occurs when $x=0$

$(-2, 1.5)$



global max of 3 when $x=-2$

can't identify a global minimum

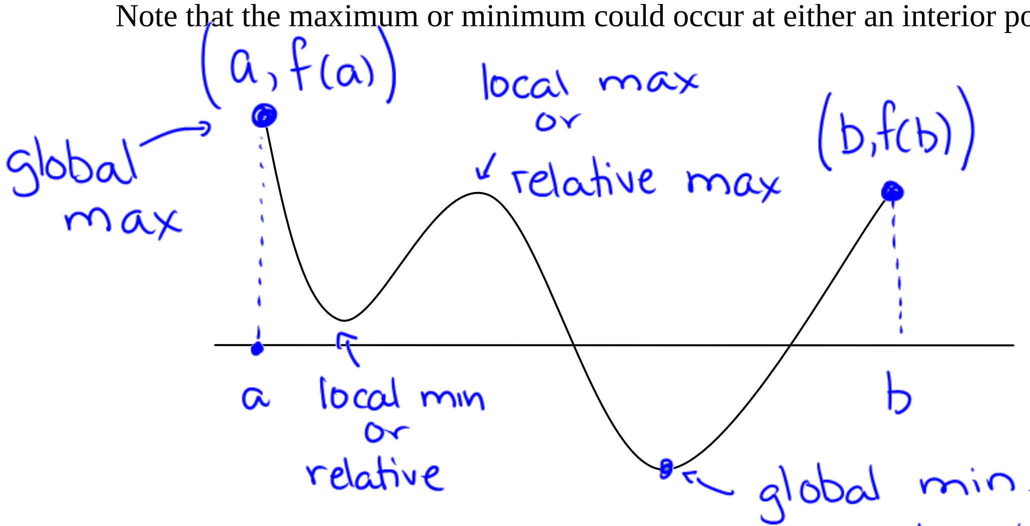
$[-2, 0)$

Extreme Value Theorem

"closed interval is one that includes the endpoints"

If f is a continuous function on a closed interval $[a,b]$, then f has both a global maximum and minimum on $[a,b]$

Note that the maximum or minimum could occur at either an interior point or at an endpoint.



f(x)

D: [a,b]

Relative (Local) Extreme Values

If c is an interior point in the domain, then $f(c)$ is a:

a) relative maximum at c iff. $f(x) \le f(c)$ for all x in some open interval containing c

b) relative minimum at c iff. $f(x) \ge f(c)$ for all x in some open interval containing c

Local Extrema also include global extrema

Where do extreme values occur?

If a function has a local extreme value at an interior point c of its domain, and if f' exists at c, then $f' = 0$

Critical Point – any point in the interior of the domain of f where either $f' = 0$ or f' is undefined

$\therefore$ Extreme values occur only at critical points or at endpoints.

* note: endpoints can be local or global max/min.

Extreme Values

Determine any extrema for the following functions on the indicated interval and where they occur. Also identify whether they are local or global extrema.

1) $f(x) = x^3 - 3x^2$ on $[-2, 5]$

Find the derivative: $f'(x) = 3x^2 - 6x$

Determine if there are any critical points:

$f'(x) = 0$

(Horizontal tangents)

$3x^2 - 6x = 0$

$3x(x-2) = 0$

$x = 0, 2$

Is $f'(x)$ undefined anywhere in the interval?

(Corners , cusps or vertical tangents)

$f'(x) = 3x^2 - 6x = \text{undefined}$

not undefined for any value of x

Determine the values of the function at then endpoints of the interval and at any critical points within the interval. Use the function values to ascertain whether there are any extrema.

	x	***Function Value***	***Conclusion***
endpoint	-2	-20	$(-2)^3 - 3(-2)^2$ global min
internal crit. points	0	0	local max
	2	-4	$(2)^3 - 3(2)^2$ local min.
endpoint	5	50	$(5)^3 - 3(5)^2$ global max

If the interval had been $(-2, 5)$, what difference(s) would this have made?

no global max or global min.

2) $f(x) = e^{-x}$ on $[-2, 2]$

$f'(x) = e^{-x} \cdot (-1)$

$f'(x) = -e^{-x}$

$-e^{-x} = 0$

no solution. in domain $[-2, 2]$

$\therefore$ no critical points

endpoints

x	y	
-2	$e^{-(-2)}$ or e^2	global max
2	$e^{-(2)}$ or $\frac{1}{e^2}$	global min

3) $f(x) = x^{2/5}$ on $[-1, 2]$

$f'(x) = \frac{2}{5} x^{-3/5}$

$f'(x) = \frac{2}{5\sqrt[5]{x^3}}$

$f'(x)$ can never $= 0$ because no x in numerator

$f'(x) =$ undefined when $x = 0$

x	y	
-1	1	← local max
0	0	← global min
2	$\sqrt[5]{2^2}$	← global max