FCP 10 Date:________________

4.3 Rational Exponents

Complete the following table:

			Conclusion
$9 = 9^1$	$\sqrt{9} \cdot \sqrt{9} =$ 9	$9^{\frac{1}{2}} \cdot 9^{\frac{1}{2}} = 9^1$	$\sqrt{9} = 9^{\frac{1}{2}}$
$25 = 25^1$	$\sqrt{25} \cdot \sqrt{25} =$ 25	$25^{\frac{1}{2}} \cdot 25^{\frac{1}{2}} = 25^1$	$\sqrt{25} = 25^{\frac{1}{2}}$
$36 = 36^1$	$\sqrt{36} \cdot \sqrt{36} =$ 36	$36^{\frac{1}{2}} \cdot 36^{\frac{1}{2}} = 36^1$	$\sqrt{36} = 36^{\frac{1}{2}}$
			$100^{\frac{1}{2}} =$
$8 = 8^1$	$\sqrt[3]{8} \cdot \sqrt[3]{8} \cdot \sqrt[3]{8} =$ 8	$8^{\frac{1}{3}} \cdot 8^{\frac{1}{3}} \cdot 8^{\frac{1}{3}} = 8^1$	$\sqrt[3]{8} = 8^{\frac{1}{3}}$
$27 = 27^1$	$\sqrt[3]{27} \cdot \sqrt[3]{27} \cdot \sqrt[3]{27} =$	$27^{\frac{1}{3}} \cdot 27^{\frac{1}{3}} \cdot 27^{\frac{1}{3}} =$	
			$64^{\frac{1}{3}} =$

Thus $x^{\frac{1}{2}} =$ $\sqrt{x}$ and $x^{\frac{1}{3}} =$ $\sqrt[3]{x}$.

$\sqrt[4]{x} = x^{\frac{1}{4}}$ $\qquad$ $\sqrt[4]{5000}$

Alternately, we could say that raising a number to the power of $\frac{1}{2}$ is the same as finding the square root of the number, and raising a number to the power of $\frac{1}{3}$ is the same as finding the cube root of the number.

index

By extension, $x^{\frac{1}{6}} =$ $\sqrt[6]{x}$ and more generally, $\boxed{x^{\frac{1}{n}} = \sqrt[n]{x}}$

radicand

be careful

$4\sqrt{x} \neq \sqrt[4]{x}$

this is a coefficient $\qquad$ index of the root.

$x^{\frac{a}{b}} = \sqrt[b]{x^a}$ or $\left(\sqrt[b]{x}\right)^a$

We also have $(8)^{\frac{2}{3}} = \left(8^{2\cdot\frac{1}{3}}\right) = \left(8^2\right)^{\frac{1}{3}} = \sqrt[3]{8^2} = \underline{\sqrt[3]{64}} = 4$

$= \left(8^{\frac{1}{3}}\right)^2 = \left(\sqrt[3]{8}\right)^2 = \underline{(2)^2} = 4$

Because rational numbers can be written in fractional or decimal form, we also have

$(32)^{.8} = (32)^{\frac{4}{5}} = \sqrt{\quad} = \_\_\_\_\_\_ = \_\_$ or

$(32)^{.8} = (32)^{\frac{4}{5}} = \left(\sqrt{\quad}\right) = \_\_\_\_\_\_ = \_\_$

eg $27^{\frac{4}{3}}$

$\sqrt[3]{27^4}$ or $\left(\sqrt[3]{27}\right)^4$

81

$(81)^{1.5} = 81^{\frac{3}{2}} = \left(\sqrt{81}\right)^3 = 81\times 9 = \boxed{\underline{729}}$

or $\sqrt{81^3}$

Generally, we can say

$$x^{\frac{m}{n}} = \sqrt[n]{x^m} = \left(\sqrt[n]{x}\right)^m$$

denominator → index of the root.

The exponent laws hold for rational exponents the same way they did for integral exponents.

1. Using exponent laws, rewrite the following with a single exponent:

a) $\left(5^{\frac{2}{3}}\right)\left(5^{\frac{7}{3}}\right)$ $= 5^{\frac{2}{3}+\frac{7}{3}}$ $= 5^{\frac{9}{3}}$ $= 5^3$	b) $\left(x^{-2}\right)\left(x^{-\frac{3}{2}}\right)$ $= x^{-\frac{4}{2}+\frac{-3}{2}}$ $= x^{-\frac{7}{2}}$ $= \frac{1}{x^{7/2}} \rightarrow \frac{1}{\sqrt{x^7}}$	c) $\frac{7^{-0.8}}{7^{-0.5}} = 7^{-.8-(-.5)}$ $= 7^{-0.3}$ $= 7^{-\frac{3}{10}}$ $= \frac{1}{\sqrt[10]{7^3}}$

d) $\dfrac{16^{1.5}}{8^{.4}}$

Convert to powers of 2

$$= \frac{(2^4)^{1.5}}{(2^3)^{.4}} = \frac{2^6}{2^{1.2}} = 2^{6-1.2} = 2^{4.8} = 2^{\frac{48}{10}} = \sqrt[10]{2^{48}} = \sqrt[5]{2^{24}}$$

e) $(81x^8)^{0.5}$

$$81^{\frac{1}{2}} \cdot x^4$$

$$9x^4$$

f) $\dfrac{3^{-\frac{1}{2}} \cdot 3^{\frac{1}{3}}}{3^{\frac{5}{6}}}$

$$\frac{3^{\frac{1}{3}}}{3^{\frac{1}{2}} \cdot 3^{\frac{5}{6}}}$$

or

$$= 3^{-\frac{1}{2}+\frac{1}{3}-\frac{5}{6}} = 3^{-\frac{3}{6}+\frac{2}{6}-\frac{5}{6}} = 3^{\frac{-6}{6}} = 3^{-1} = \frac{1}{3}$$

g) $\left[(x^{-2})(x^{\frac{2}{3}})\right]^{-\frac{1}{4}}$

$$= x^{\frac{2}{4}} \cdot x^{-\frac{1}{6}} = x^{\frac{6}{12}} \cdot x^{-\frac{2}{12}} = x^{\frac{4}{12}} = x^{\frac{1}{3}} \text{ or } \sqrt[3]{x}$$

h) $\left(\dfrac{x^4}{16}\right)^{-\frac{3}{4}}$

$$= \frac{x^{-3}}{16^{-\frac{3}{4}}} = \frac{16^{\frac{3}{4}}}{x^3} = \frac{(\sqrt[4]{16})^3}{x^3} = \frac{2^3}{x^3} = \frac{8}{x^3}$$

i) $\left[\left(\dfrac{8}{27}\right)^2\left(\dfrac{8}{27}\right)^{-4}\right]^{-\frac{1}{3}}$

$$= \left[\left(\frac{8}{27}\right)^{-2}\right]^{-\frac{1}{3}} = \left(\frac{8}{27}\right)^{\frac{2}{3}} = \frac{(\sqrt[3]{8})^2}{(\sqrt[3]{27})^2} = \frac{2^2}{3^2} = \frac{4}{9}$$

2. Moneybags invests $8000 in a fund which earns 9.6% per year. The bank provides an electronic update on the value of the fund every 3 months using the formula $A = 8000(1.096)^{\frac{x}{4}}$ where x represents the number of 3 month (quarter of a year) periods that have passed.

a) How are the interest rate of 9.6% and the value of 1.096 in the formula related?

b) What is the value of the investment after 9 months?

c) What is the value of the investment after 5 years?

d) What amount of money would Moneybags have had to invest 1 year ago so that she would now have $8000?

Section 4.3 Extra Practice

1. Use the exponent laws to simplify each expression.

a) $\left(x^{\frac{1}{2}}\right)\left(x^{\frac{7}{2}}\right)$

b) $\left(3m^4\right)\left(m^{\frac{1}{4}}\right)$

c) $[(x^{1.5})(x^{2.5})]^{0.5}$

d) $\left(\frac{5x^3}{20x}\right)^{\frac{1}{2}}$

e) $\left(x^{\frac{2}{3}}y^{\frac{4}{3}}\right)^3$

2. Simplify each expression. State the answer using positive exponents.

a) $\left(y^{-2}\right)\left(y^{\frac{5}{2}}\right)$

b) $\left(-8x^{-6}\right)^{\frac{1}{3}}$

c) $\dfrac{\left(x^3\right)^{\frac{1}{2}}}{\left(x^{\frac{5}{2}}\right)^{\frac{1}{5}}}$

d) $\left(\dfrac{x^{\frac{1}{4}}}{16x^{\frac{3}{4}}}\right)^{\frac{1}{2}}$

e) $\left(x^{\frac{1}{3}}y^{\frac{4}{5}}\right)^0\left(x^{\frac{1}{3}}\right)^6$

3. Evaluate without using a calculator. Leave each answer as a rational number.

a) $\dfrac{5^{-2}}{125^{\frac{1}{3}}}$

b) $\dfrac{9^{\frac{3}{2}}}{27^2}$

c) $\left(8^{\frac{2}{3}}\right)\left(16^{\frac{3}{2}}\right)$

d) $\left(3^{-2}\right)^{\frac{-5}{2}}$

e) $\left(125^{\frac{-1}{3}}\right)^2$

4. Evaluate using a calculator. Give the result to four decimal places, if necessary.

a) $(7^{1.2})^{-3}$

b) $\left(4^3\right)\left(4^{\frac{3}{2}}\right)$

c) $(7^3)^{\frac{2}{3}}$

d) $\left(\dfrac{6^2}{3^3}\right)^{\frac{1}{3}}$

e) $\left[\dfrac{3^2}{(-3)^4}\right]^{\frac{1}{2}}$

5. The growth of 5000 bacterium cells in a lab can be modelled using the expression $N = 5000(1.5)^{\frac{h}{40}}$, where N is the number of bacteria after h hours.

a) What does the value 1.5 in the expression tell you?

b) How many bacteria are there after 40 h?

c) How many more bacteria are there after 3 h?

d) What does $h = 0$ indicate?

---

1. a) x^4 **b)** $3m^{\frac{17}{4}}$ **c)** x^2 **d)** $\frac{x}{2}$ **e)** x^2y^4 **2. a)** $y^{\frac{1}{2}}$ **b)** $\frac{-2}{x^2}$ **c)** x **d)** $\dfrac{1}{4x^{\frac{1}{4}}}$ **e)** x^2

3. a) $5^{-3} = \frac{1}{125}$ **b)** $3^{-3} = \frac{1}{27}$ **c)** $2^8 = 256$ **d)** $3^5 = 243$ **e)** $\left(\frac{1}{5}\right)^2 = \frac{1}{25}$ **4. a)** $7^{-3.6} = 0.0009$ **b)** $4^{\frac{9}{2}} = 512$ **c)** $7^2 = 49$ **d)** $\dfrac{6^{\frac{2}{3}}}{3} = 1.1006$ **e)** $3^{-1} = 0.3333$

5. a) The number of bacteria increases by 1.5 times every 40 h.
b) 7500. There are 7500 bacteria after 40 h.
c) 5154.385; 5154.385 – 5000 = 154.385. There are approximately 154 more bacteria after 3 h.
d) Example: The value $h = 0$ indicates the starting population of 5000 bacteria.