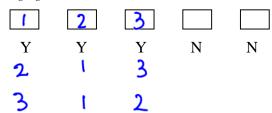
11.2 Combinations

Investigate

1. If 5 sprinters compete in a race, how many different ways can the medals for first, second and third place, be awarded?

2. If 5 sprinters compete in a race and the fastest 3 qualify for the relay team, how many different relay teams can be formed?

Visualize the 5 sprinters below. Since 3 will qualify for the relay team and 2 will not, consider the number of ways of arranging 3 Y's and 2 N's.



Does the order of finish for the fastest three matter here?

This is an example of a **combination** of $\underline{5}$ objects taken $\underline{3}$ at a time.

Combinations

- An *unordered* arrangement of distinct objects is called a *combination*.
- The number of combinations of n distinct objects taken r at a time is

$$_{n}C_{r} = \binom{n}{r} = \frac{nP_{r}}{r!} = \frac{n!}{r!(n-r)!}$$
 with $n \ge r \ge 0$.

Permutations
$$n!$$
 $nPr = \frac{n!}{(n-r)!}$
 $n Cr = \frac{n!}{r!(n-r)!}$
"n choose r"

 $49(7 = \frac{49!}{7!42!} = 85,9 00,584$

Example 1:

a) How many different committees of 3 people can be formed from 7 people?

order does not matter
$$7C_3 = \frac{7!}{3!4!}$$

b) How many different committees of 3 people can be formed from 7 people if the first person selected serves as the chairperson, the second as the treasurer, and the third as the secretary?

order does matter
$$7P_3 = \frac{7!}{4!}$$

c) If the group of 7 people consists of 3 males and 4 females, how many different committees of 3 people can be formed with 1 male and 2 females?

Think: you must choose 1 male out of the group of 3 males and 2 females out of the group of 4 females.

I make and 2 females
$$\frac{4!}{2!2!} = \frac{24}{4}$$

$$3C_1 \times 4C_2 \times 6 = 18$$

d) If the group of 7 people consists of 3 males and 4 females, how many different committees of 3 people can be formed with at least one male on the committee?

a) Evaluate:
$$\frac{100!}{3!97!}$$
 =

b) Solve:
$${}_{n}C_{2} = 21$$
 $2! (n-2)!$

$$\frac{2!(n-2)!}{n \times (n-1) \times (n-2)!} = 21$$

$$\frac{2(n-2)!}{2(n-2)!} = 21$$

$$\frac{n^2-n}{n^2-n-42} = 42$$

$$\frac{n^2-n-42}{(n-7)(n+6)=0} = 7$$

Example 3:

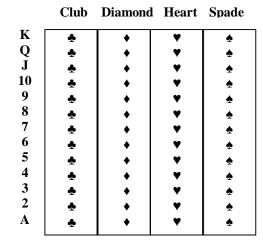
A standard deck of 52 playing cards consists of 4 suits (spaces, hearts, diamonds, and clubs) of 13 cards each.

a) How many different 5-card hands can be formed?

52(5

b) How many different 5-card hands can be formed that consist of all hearts?

13 (5



- c) How many different 5-card hands can be formed that consist of all face cards? K,Q,
- d) How many different 5-card hands can be formed that consist of 3 hearts and 2 spades?

- e) How many different 5-card hands can be formed that consist of exactly $\frac{3 \text{ hearts}}{3 \text{ kearts}}$ and $\frac{2 \text{ not hearts}}{3 \text{ kearts}}$
- f) How many different 5-card hands can be formed that consist of at least 3 hearts?

3H and 2non or 4H and 1 non or 5H and 0 non
$$3C_3 \times 39C_2 + 3C_4 \times 39C_1 + 3C_5 \times 39C_6$$
286 x 741 + 715 x 39 + 1287 x 1
241 098

Example 4

A basketball coach has five guards and seven forwards on his basketball team.

a) In how many different ways can be select a starting team of two guards and three forwards?

5 (2 x 7 (3

b) How many starting teams are there if the star player, who plays guard, must be included?

1 Star and 1 other guard and 3 forwards

1 $C_1 \times 4C_1 \times 7C_3$

Example 5

a) Explain in words why $_{10}C_7$ is the same as $_{10}C_3$

Explain in words why ${}_{10}C_7$ is the same as ${}_{10}C_3$ choosing which 7 people to give candy to is the same as choosing which 3 people do not receive

b) Use factorial notation to show $_{10}C_7 = _{10}C_3$

 $\frac{10!}{7! \cdot 3!} = \frac{10!}{3! \cdot 7!}$

c) Prove the identity: ${}_{n}C_{r} = {}_{n}C_{n-r}$

 $\frac{n!}{r!(n-r)!} = \frac{n!}{(n-r)!(n-[n-r])!}$ $=\frac{n!}{(n-r)!(r)!}$

Example 6

During a basketball tournament, all players shake hands with each other at the end of the last game. If 300 handshakes were exchanged, how many players were at the tournament?

 $nC_2 = 30$ $\frac{n!}{2!(n-2)!} = 300$

 $\frac{(n)(n-1)(n-2)!}{2(n-2)!} = 300$ n(n-1) = 600