Stretching Graphs of Functions

1. Comparing the graphs of $y = f(x)$ and ~~$cy = f(x)$~~ $y = c\,f(x)$

a) Complete the following tables of values by first rewriting the equation with the indicated substitution and then solving the equation for *y*. The first one is completed for you.

$y = x^2$

x	y
–3	9
–2	4
–1	1
0	0
1	1
2	4
3	9

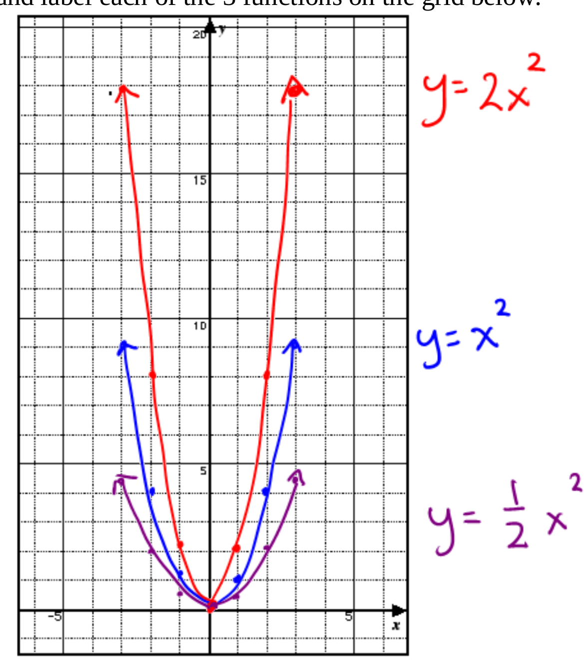
$y = 2x^2$

x	y
–3	18
–2	8
–1	2
0	0
1	2
2	8
3	18

$y = \frac{1}{2}x^2$

x	y
–3	4.5
–2	2
–1	.5
0	0
1	.5
2	2
3	4.5

b) Use the tables of values to graph and label each of the 3 functions on the grid below.



c) How can each of the following graphs be obtained from the graph of $y = x^2$?

i) $y = 2x^2$ v. stretch by a factor of 2 (all y-coordinates are ×2)

ii) $y = \frac{1}{2}x^2$ v. stretch by a factor of $\frac{1}{2}$ (all y-coords are $\times\frac{1}{2}$)

d) In general, how is the graph of $y = ax^2$ obtained from the graph of $y = x^2$

i) when $a > 1$?

a stretch that increases size (expansion)

ii) when $0 < a < 1$?

a stretch that decreases size (compression)

Summary:

• If $a > 1$, the graph of $y = af(x)$ is obtained when the graph of $y = f(x)$ undergoes a vertical stretch (expansion) by a factor of *a*.

• If $0 < a < 1$, the graph of $y = af(x)$ is obtained when the graph of $y = f(x)$ undergoes a vertical stretch (compression) by a factor of *a*.

Remember that the *y*–values of $y = af(x)$ are obtained by multiplying each *y*–value of $y = f(x)$ by the factor *a*.

What happens if $a < 0$? (negative) eg $y = -2x^2$

In general, if $a < 0$, the graph of $y = af(x)$ is obtained when the graph of $y = f(x)$ undergoes a vertical stretch by a factor of *a*, along with a vertical reflection.

NB: multiply every y-coord by -2 to find new y-coord

Note: The notation $\frac{y}{a} = f(x)$ is also used instead of $y = af(x)$ to emphasize that the parameter ***a*** involves a stretch in the *y*–direction: i.e., a *vertical* stretch.

$y = 2f(x)$ is same as $\frac{y}{2} = f(x)$ or $\frac{1}{2}y = f(x)$

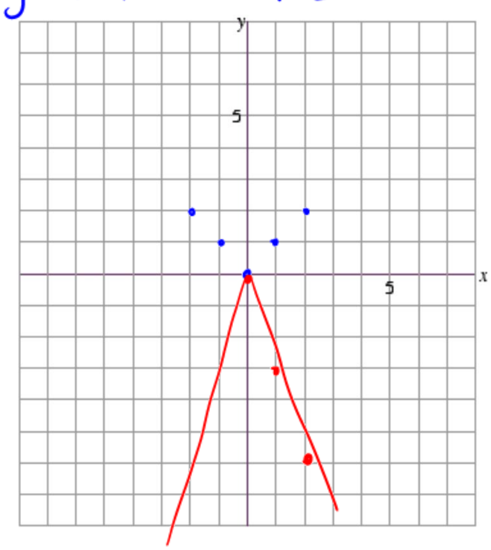
Example 1:

Sketch the graph of $y = -3|x|$.

We can obtain the graph of $y = -3|x|$ from the graph of $y = |x|$ through two transformations:

a) reflection

b) v. stretch by a factor of 3



$(2,2) \rightarrow (2,-6)$

$(1,1) \rightarrow (1,-3)$

$(0,0) \rightarrow (0,0)$

2.	**Comparing the graphs of $y = f(x)$ and $y = f(ax)$**

a) Complete the following tables of values. The first one is completed for you.

$y = \sqrt{x}$

x	y
16	4
9	3
4	2
1	1
0	0

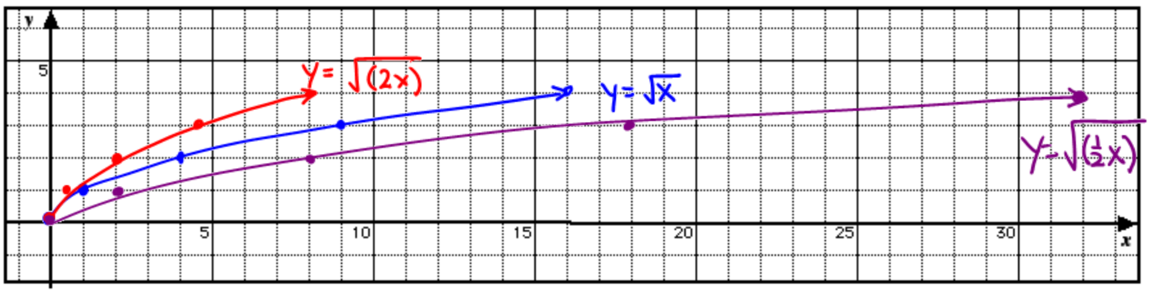
$y = \sqrt{(2x)}$

x	y
8	4
4.5	3
2	2
0.5	1
0	0

$y = \sqrt{(0.5x)}$ or $y = \sqrt{\frac{1}{2}x}$

x	y
32	4
18	3
8	2
2	1
0	0

b) Use the tables of values to graph and label each of the 3 functions on the grid below.



c) How can each of the following graphs be obtained from the graph of $y = \sqrt{x}$?

i) $y = \sqrt{(2x)}$ horizontal stretch by a factor of $\frac{1}{2}$

ii) $y = \sqrt{(0.5x)}$ h. stretch by a factor of $\frac{2}{1}$

$y = \sqrt{\frac{1}{2}x}$

d) In general, how is the graph of $y = \sqrt{(bx)}$ obtained from the graph of $y = \sqrt{x}$

i) when $b > 1$? h. compression (stretch) by a factor of $\frac{1}{b}$

ii) when $0 < b < 1$?

h. stretch (expansion) by a factor of $\frac{1}{b}$.

Summary:

- If $b > 1$, the graph of $y = f(bx)$ is obtained when the graph of $y = f(x)$ undergoes a h. stretch (compression) by a factor of $\frac{1}{b}$. the reciprocal
- If $0 < b < 1$, the graph of $y = f(bx)$ is obtained when the graph of $y = f(x)$ undergoes a h. stretch (expansion) by a factor of $\frac{1}{b}$. the reciprocal

eg $b = 2$ $y = f(2x)$ stretch by $\frac{1}{2}$

$b = \frac{1}{2}$ $y = f(\frac{1}{2}x)$ stretch by $\frac{2}{1}$

Notice from your tables that for $y=\sqrt{(2x)}$ to have the same *y*–values as $y=\sqrt{x}$, the corresponding *x*–values of $y=\sqrt{(2x)}$ must be divided by the factor 2.

Thus in general, for $y = f(bx)$ to have the same *y*–values as $y = f(x)$, the corresponding *x*–values of $y = f(bx)$ must be divided by the factor *b*.

In other words, if $b>1$, it takes "less *x*" to do the job of building the function $y = f(bx)$, so we have a horizontal *compression* of $y = f(x)$. * you need a stretch to compensate for the coefficient you put in.

Also, if $0<b<1$, it takes "more *x*" to do the job of building the function $y = f(bx)$, so we have a horizontal *expansion* of $y = f(x)$.

What happens if $b<0$?

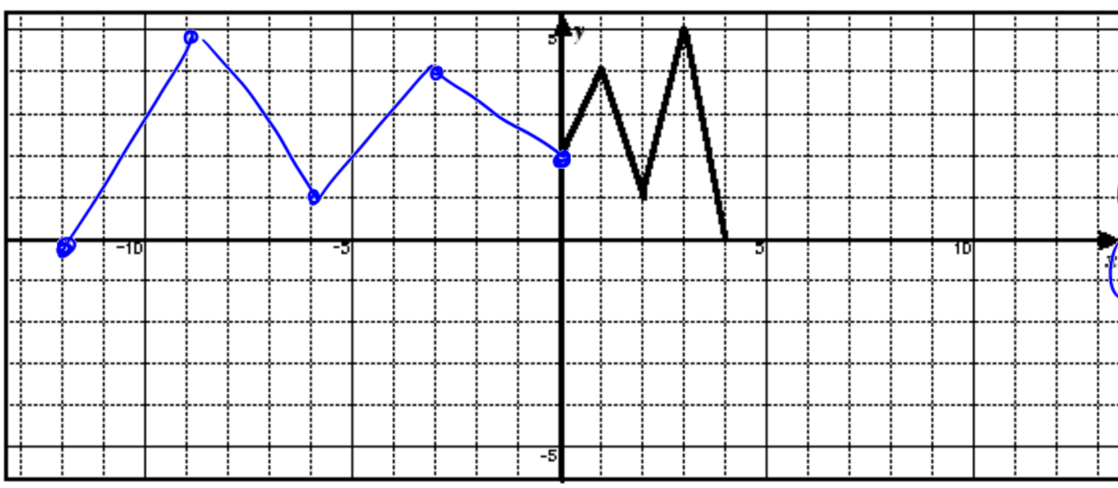
In general, if $b<0$, the graph of $y = f(bx)$ is obtained when the graph of $y = f(x)$ undergoes a horizontal stretch by a factor of $\frac{1}{b}$, along with a reflection in the y-axis.

or as coordinate, x is multiplied by the negative reciprocal.

Example 2:

The grid below contains the graph of a function $y = f(x)$. Sketch the graph of $y = f(-\frac{1}{3}x)$.

multiply each x by -3



$(1,4) \rightarrow (-3,4)$

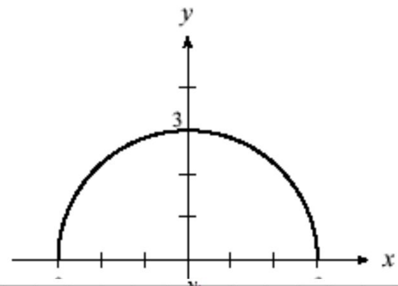
$(2,1) \rightarrow (-6,1)$

$(3,5) \rightarrow (-9,5)$

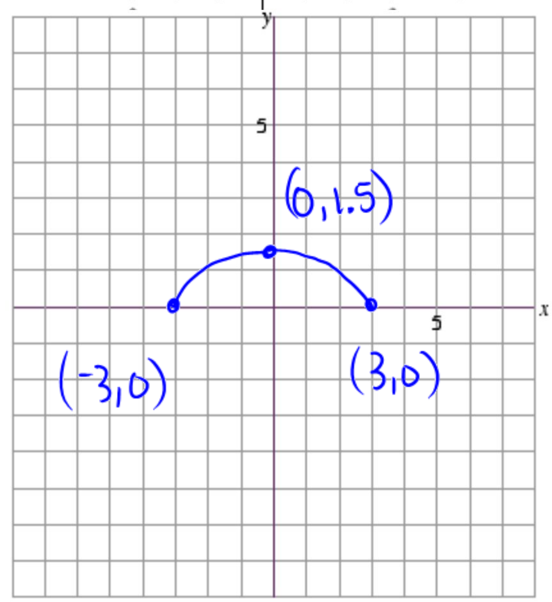
$(4,0) \rightarrow (-12,0)$

Example 3:

The graph of $y = \sqrt{9 - x^2}$ is shown to the right.



Sketch the graph of $2y = \sqrt{9 - x^2}$.



$y = \frac{1}{2}\sqrt{9 - x^2}$

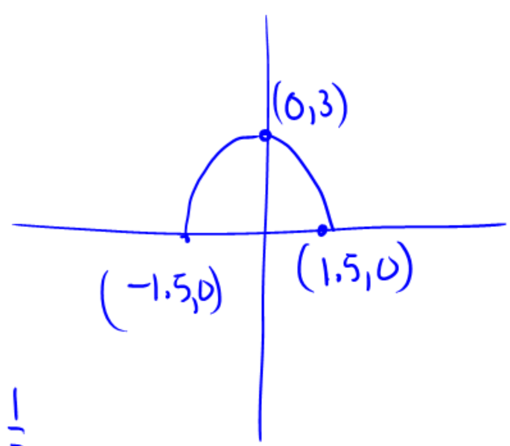
v. stretch by $\frac{1}{2}$

Sketch the graph of

$y = \sqrt{9 - (2x)^2}$

$x \to 2x$

h. stretch (compression) by a factor of $\frac{1}{2}$.



$y = f(x)$	v. stretch by 2	h. stretch by 2
$y = x^2$	$y = 2 \cdot x^2$	$y = \left(\frac{1}{2}x\right)^2$
$y = \frac{1}{x}$	$y = 2 \cdot \frac{1}{x}$ or $y = \frac{2}{x}$	$y = \frac{1}{\left(\frac{1}{2}x\right)}$
$y = (x+3)^2$	$y = 2(x+3)^2$	$y = \left(\frac{1}{2}x + 3\right)^2$