Math 12 Pre-Calculus
Warm-Up

$f(x) \xrightarrow{\text{left 3 units}} f(x+3)$

$(-3,5)$

1. The point $(-3, 5)$ lies on the graph of $y = f(x+3)$. What must be a point on the graph of

   a) $y = f(x)$ $(0,5)$

   $f(x+3-5)$

   $f(x+3) \rightarrow f(x-2)$

   $(-3,5)$ $(2,5)$

   right 5 units

   b) $y = f(x-2)$

   $f(x) \xrightarrow{\text{right 2}} f(x-2)$

   $(0,5)$ $(2,5)$

   c) $y = f(x) + 4$ $(0,9)$

   up 4

   d) $y = f(x+4) - 7$ $(-4,-2)$

   down 7, left 4

2. The point (h, k) lies on the graph of $y = f(x)$. What must be a point on the graph of

   a) $y = f(x+1)$ $(h-1, k)$

   left 1

   b) $y = f(x-2)$ $(h+2, k)$

   right 2

   c) $y = f(x) + 4$ $(h, k+4)$

   up 4

   d) $y = f(x+4) - 7$ $(h-4, k-7)$

3. A function has equation $y = (x-3)(x-2)(x+1)$. Write the equation of the function that is translated right 2 units and up 3 units.

   $x-2$ $+3$

   $y = (x-3-2)(x-2-2)(x+1-2)+3$

   $y = (x-5)(x-4)(x-1) + 3$

1.2 Reflections and Stretches

1. Comparing the graphs of $y = f(x)$ and $y = -f(x)$

a) Complete the second table of values. The first one is completed for you.

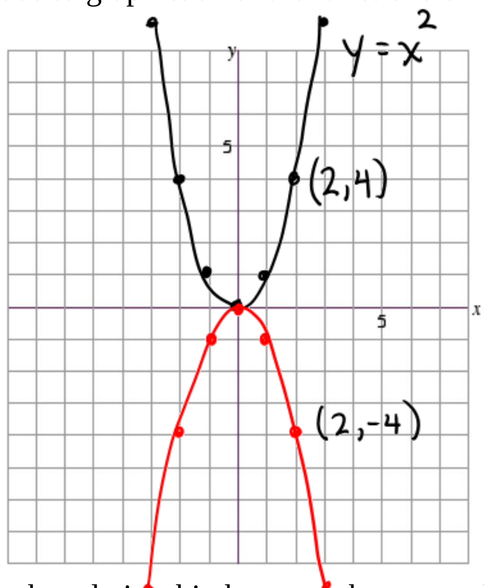
$y = x^2$

x	y
–3	9
–2	4
–1	1
0	0
1	1
2	4
3	9

$y = -x^2$

x	y
–3	-9
–2	-4
–1	-1
0	0
1	-1
2	-4
3	-9

b) Use the tables of values above to graph each of the functions on the grid below.



$y=x^2 \longrightarrow y=-x^2$

$(2,4) \longrightarrow (2,-4)$

c) For the two graphs, what is the relationship between the *y*–coordinates of points that have the same *x*–coordinates? y- coordinates are opposites, the points are "reflected"

d) Describe how the graph of $y = x^2$ is related to the graph of $y = -x^2$. (In other words, what happens to the graph of $y = x^2$ when a negative sign is placed in front of the term x^2?) reflected

e) In general, the graph of $y = -f(x)$ is a vertical reflection of the graph of $y = f(x)$ in the x-axis. (x-axis is the "mirror")

Note: The notation $-y = f(x)$) is sometimes used instead of $y = -f(x)$ to emphasize that the reflection involves a reversal of y–coordinates.

$-y = f(x)$
same as
$y = -f(x)$

$x \to -x$

2. Comparing the graphs of $y = f(x)$ and $y = f(-x)$.

a) Complete the second table of values. The first one is completed for you.

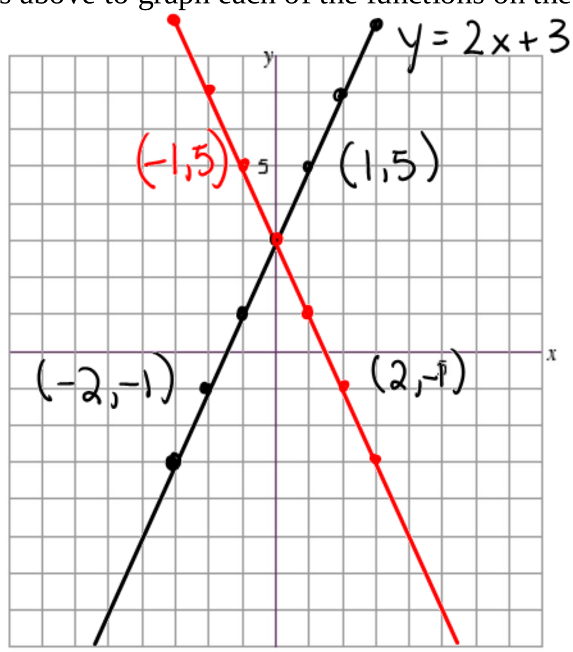
$y = 2x + 3$

x	y
–3	–3
–2	–1
–1	1
0	3
1	5
2	7
3	9

$y = 2(-x) + 3$

x	y
–3	9
–2	7
–1	5
0	3
1	1
2	-1
3	-3

b) Use the tables of values above to graph each of the functions on the grid below.



c) For the two graphs, what is the relationship between the x–coordinates of points that have the same y–coordinates? they are opposites

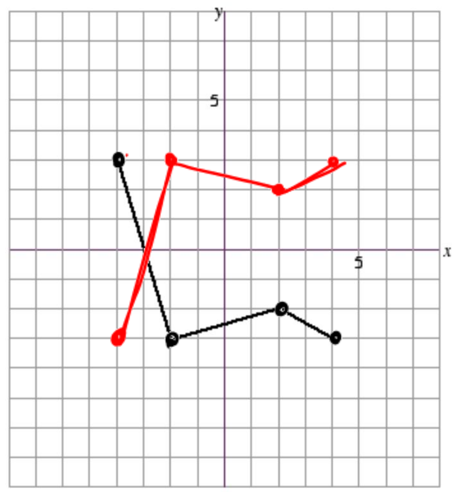
d) Describe how the graph of $y = 2x + 3$ is related to the graph of $y = 2(-x) + 3$. (In other words, what happens to the graph of $y = 2x + 3$ when a negative sign is placed in front of the term x?) horizontal reflection

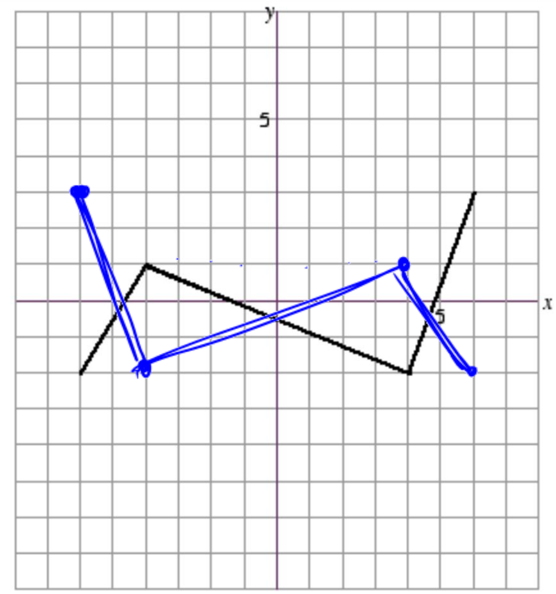
e) In general, the graph of $y = f(-x)$ is a horizontal reflection of the graph of $y = f(x)$ in the y-axis .

Example 1:
In each case, the graph of a function $y = f(x)$ is shown. Sketch the graph of the reflected function indicated.

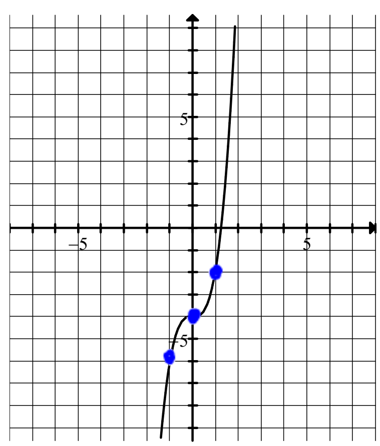
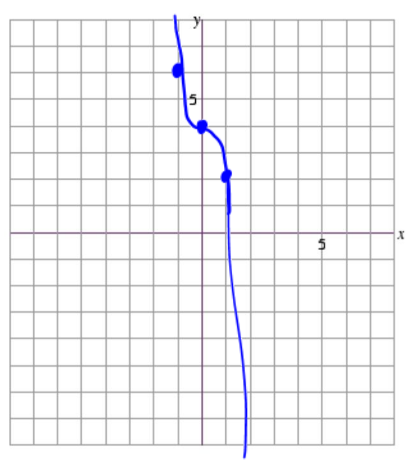
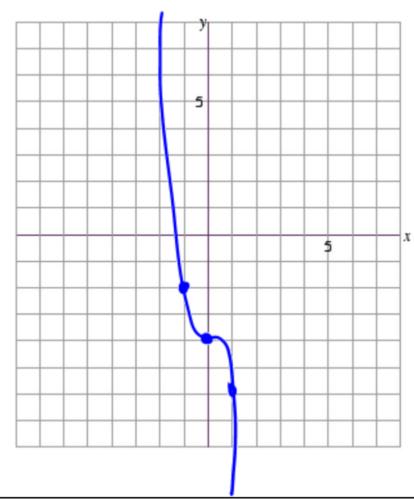
a) $y = -f(x)$ ← vertical

b) $y = f(-x)$ ← horizontal





Example 2:
Using the graph of $f(x) = 2x^3 - 4$ on the left, sketch each of the indicated graphs.

$f(x) = 2x^3 - 4$	$y = -f(x)$	b) $y = f(-x)$
		

Example 3:
Given the function $f(x) = \underbrace{2x^3 - 4}$, write equations for

a) $y = -f(x)$

$y = -(2x^3 - 4)$

$y = -2x^3 + 4$

b) $y = f(-x)$

$y = 2(-x)^3 - 4$

$= 2(-x^3) - 4$

$y = -2x^3 - 4$

Stretching Graphs of Functions

$y=f(x)$ and $y=af(x)$

1. Comparing the graphs of $y = f(x)$ and $cy = f(x)$

a) Complete the following tables of values by first rewriting the equation with the indicated substitution and then solving the equation for *y*. The first one is completed for you.

$y = x^2$

x	y
–3	9
–2	4
–1	1
0	0
1	1
2	4
3	9

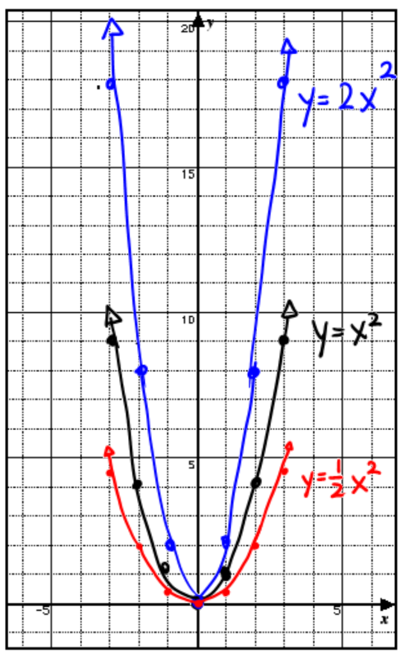
$y = 2x^2$

x	y
–3	18
–2	8
–1	2
0	0
1	2
2	8
3	18

$y = \frac{1}{2}x^2$

x	y
–3	4.5
–2	2
–1	0.5
0	0
1	0.5
2	2
3	4.5

b) Use the tables of values to graph and label each of the 3 functions on the grid below.



c) How can each of the following graphs be obtained from the graph of $y = x^2$?

i) $y = 2x^2$ taller, y-coords are twice as big.

ii) $y = \frac{1}{2}x^2$ Shorter, y-coords are $\frac{1}{2}$ as big.

d) In general, how is the graph of $y = ax^2$ obtained from the graph of $y = x^2$

i) when $a > 1$? stretched / expanded

ii) when $0 < a < 1$? squashed / compressed

Summary:

- If $a > 1$, the graph of $y = af(x)$ is obtained when the graph of $y = f(x)$ undergoes a vertical expansion by a factor of *a*.
- If $0 < a < 1$, the graph of $y = af(x)$ is obtained when the graph of $y = f(x)$ undergoes a vertical compression by a factor of *a*.

Remember that the *y*–values of $y = af(x)$ are obtained by multiplying each *y*–value of $y = f(x)$ by the factor *a*.

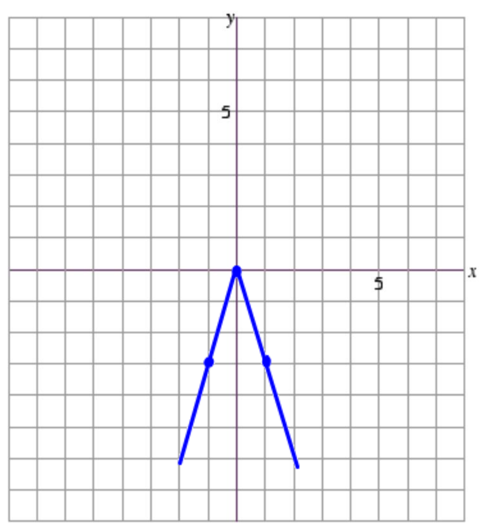
What happens if $a < 0$?
In general, if $a < 0$, the graph of $y = af(x)$ is obtained when the graph of $y = f(x)$ undergoes a vertical expansion/compression by a factor of *a*, along with a reflection in x-axis.

Note: The notation $\frac{y}{a} = f(x)$ is also used instead of $y = af(x)$ to emphasize that the parameter ***a*** involves a stretch in the *y*–direction: i.e., a *vertical* stretch.

coefficient "a" = -3

Example 1:
Sketch the graph of $y = -3|x|$.
We can obtain the graph of $y = -3|x|$ from the graph of $y = |x|$ through two transformations:

a) reflection
b) expanded by a factor of 3



$y=\|x\|$		$y=-3\|x\|$
(-1,1)	$\rightarrow$	(-1,-3)
(0,0)	$\rightarrow$	(0,0)
(1,1)	$\rightarrow$	(1,-3)

multiplied each y-coord by -3

do 1.2b #5,6

2. Comparing the graphs of $y = f(x)$ and $y = f(ax)$

a) Complete the following tables of values. The first one is completed for you.

$y = \sqrt{x}$

x	y
16	4
9	3
4	2
1	1
0	0

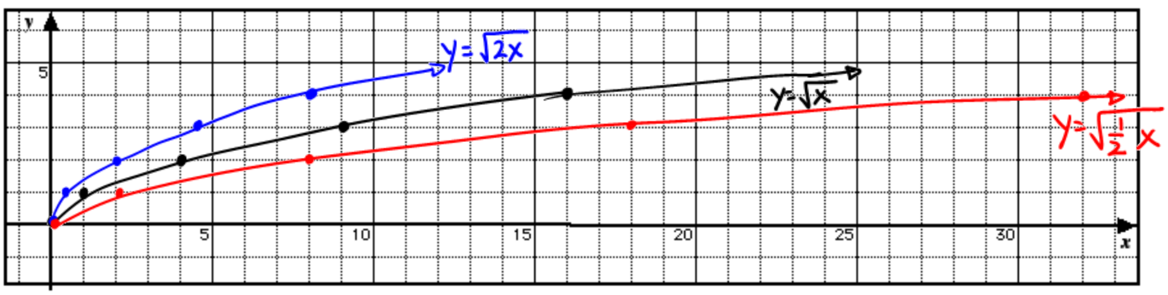
$y = \sqrt{(2x)}$

x	y
8	4
4.5	3
2	2
0.5	1
0	0

$y = \sqrt{(0.5x)}$ $y=\sqrt{\frac{1}{2}x}$

x	y
32	4
18	3
8	2
2	1
0	0

b) Use the tables of values to graph and label each of the 3 functions on the grid below.



c) How can each of the following graphs be obtained from the graph of $y = \sqrt{x}$?

i) $y = \sqrt{(2x)}$ compressed horizontally

ii) $y = \sqrt{(0.5x)}$ stretched horizontally

d) In general, how is the graph of $y = \sqrt{(bx)}$ obtained from the graph of $y = \sqrt{x}$

i) when $b > 1$?

eg. $y=\sqrt{2x}$ compression

ii) when $0 < b < 1$?

eg $y=\sqrt{\frac{1}{2}x}$ expansion

$\left.\begin{matrix} \text{compression} \\ \text{expansion} \end{matrix}\right\}$ by a factor of $\frac{1}{b}$ (reciprocal)

Summary:

- If $b > 1$, the graph of $y = f(bx)$ is obtained when the graph of $y = f(x)$ undergoes a ___horizontal___ ___compression___ by a factor of ___$\frac{1}{b}$___.
- If $0 < b < 1$, the graph of $y = f(bx)$ is obtained when the graph of $y = f(x)$ undergoes a ___horizontal___ ___expansion___ by a factor of ___$\frac{1}{b}$___ .

Notice from your tables that for $y = \sqrt{(2x)}$ to have the same *y*–values as $y = \sqrt{x}$, the corresponding *x*–values of $y = \sqrt{(2x)}$ must be divided by the factor 2.

Thus in general, for $y = f(bx)$ to have the same *y*–values as $y = f(x)$, the corresponding *x*–values of $y = f(bx)$ must be divided by the factor *b*.

In other words, if $b > 1$, it takes "less *x*" to do the job of building the function $y = f(bx)$, so we have a horizontal *compression* of $y = f(x)$.

$y=f(x)$ $\quad$ $y=f(2x)$

$(16,4)$ $\quad$ $(8\ \ ,4)$

Also, if $0 < b < 1$, it takes "more *x*" to do the job of building the function $y = f(bx)$, so we have a horizontal *expansion* of $y = f(x)$.

What happens if $b < 0$? (negative) eg $y=\sqrt{-2x}$

In general, if $b < 0$, the graph of $y = f(bx)$ is obtained when the graph of $y = f(x)$ undergoes a horizontal expansion/compression by a factor of $\frac{1}{b}$, along with a reflection in y-axis.

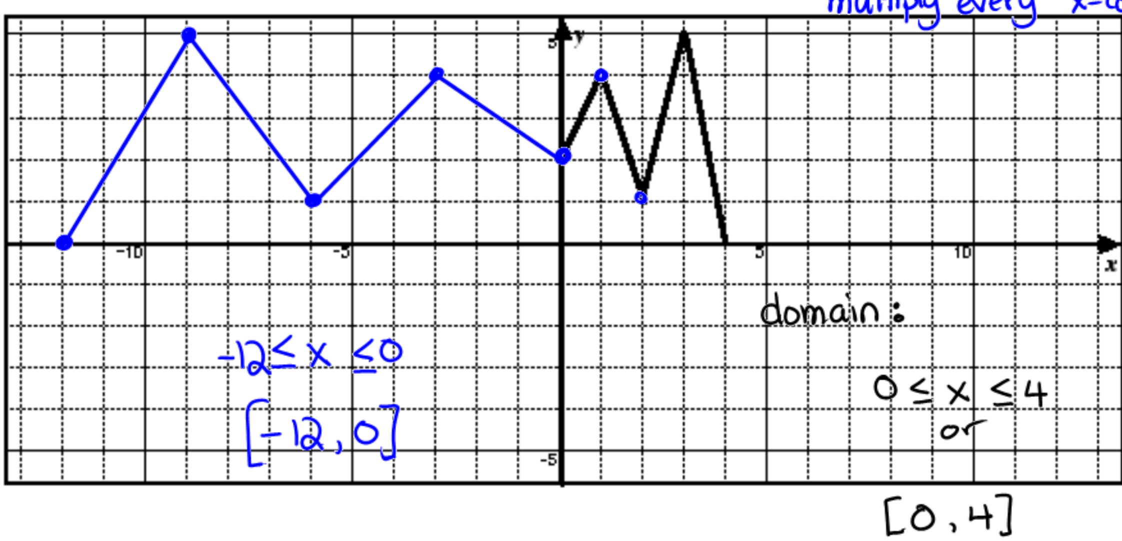
Example 2:

The grid below contains the graph of a function $y = f(x)$. Sketch the graph of $y = f(-\frac{1}{3}x)$.

reflection in y-axis

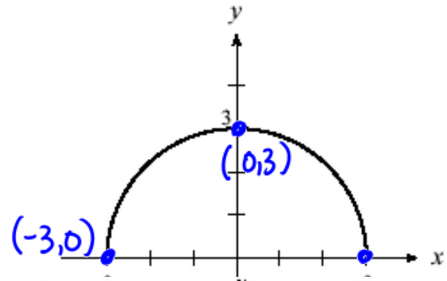
expand by factor of 3

multiply every x-coord by -3



Example 3:

The graph of $y = \sqrt{9 - x^2}$ is shown to the right.



Sketch the graph of $2y = \sqrt{9 - x^2}$.

$$\frac{2y}{2} = \frac{\sqrt{9-x^2}}{2}$$

$$y = \frac{\sqrt{9-x^2}}{2} \quad \text{or} \quad y = \frac{1}{2}\sqrt{9-x^2}$$

$\boxed{\text{V. compress by } \tfrac{1}{2}}$