11.1 Warmup

1. A café has a lunch special consisting of an egg or a ham sandwich (E or H); milk, juice, or coffee (M, J, or C); and yogurt or pie for dessert (Y or P).

a) One item is chosen from each category. List all possible meals or draw a tree diagram to represent all possible meals.

EMY	EJY	ECY	HMY	HJY	HCY
EMP	EJP	ECP	HMP	HJP	HCP

b) How many possible meals are there?

$2 \times 3 \times 2 = 12$

c) How can you determine the number of possible meals without listing all of them?

multiplying - fundamental counting principle.

2. The cafe also features ice cream in 24 flavours. You can order regular, sugar or waffle cones. Suppose you order a double cone with two scoops of ice cream. How many different double cones are possible?

if C,V is considered same as V,C

$\frac{24 \times 24 \times 3}{2}$ ← ÷2 because CV is same as VC.

3. How many different 2-digit numbers are there?

$\underline{9} \times \underline{10} = 90$ possible numbers.

1,...9 0....9

The Fundamental Counting Principle
If one item can be selected in *m* ways, and for each way a second item can be selected in *n* ways, then the two items can be selected in $m \times n$ ways.

11.1A Permutations Involving Distinct Objects

1. Two letters, A and B, can be written in two different orders: AB and BA. These are *permutations* of A and B. The arrangement of objects in a line is called a permutation, and the order of the objects is important.

permutations - ordered arrangements

a) List all the permutations of 3 letters A, B, and C.

ABC BAC BCA
ACB CAB CBA

3 2 1

$\boxed{3 \times 2 \times 1}$

b) How many permutations are there? 6

c) List all the permutations of the 4 letters A, B, C, and D

ABCD ACBD ADBC BACD BCAD BDAC
ABDC ACDB ADCB BADC BCDA BDCA

d) How many permutations are there? 24

$\boxed{4 \quad 3 \quad 2 \quad 1}$

e) Predict the number of permutations of 5 letters A, B, C, D, and E. $120 = 5 \times 4 \times 3 \times 2 \times 1$

f) How many different ways can 6 people be arranged in a line? $720 = 6 \times 5 \times 4 \times 3 \times 2 \times 1$

How many different ways can 7 different books be arranged on a shelf? $5040 = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

How many permutations of letters are there of the letters of the word PROVE ? $120 = 5 \times 4 \times 3 \times 2 \times 1$

Factorial Notation The symbol **!** is used in mathematics to denote the factorial operation.

0! = 1

1! = 1

2! = 2

3! = 6

4! = $4 \times 3 \times 2 \times 1 = 24$

5! = $5 \times 4 \times 3 \times 2 \times 1 = 120$

n! = $n \times (n-1) \times (n-2) \times \ldots \times 3 \times 2 \times 1$

n must be a whole number

2. Consider the letters A, B, C, D and E. Instead of using all the letters to form permutations, we could use fewer letters. For example, DB is a 2-letter permutation of these 5 letters.

a) List all the different 2-letter permutations of the 5 letters A, B, C, D, and E.

AB AC AD AE DA DB DC DE
BA BC BD BE EA EB EC ED
CA CB CD CE

5 4

b) How many different 3-letter permutations are there?

60 permutations.

5 4 3

3. In a row with 7 students, how many possible arrangements are there for the first 3 people in the row?

Fundamental counting principle : $7 \times 6 \times 5$

Notation: ${}_7P_3 = \frac{7!}{(7-3)!} = \frac{7 \times 6 \times 5 \times \cancel{4} \times \cancel{3} \times \cancel{2} \times \cancel{1}}{\cancel{4} \times \cancel{3} \times \cancel{2} \times \cancel{1}}$

choices uses 3 of choices

Permutations with distinct objects   • An ordered arrangement of objects is called a permutation   • The number of permutations of n distinct objects is $n!$ - if you use all of them   • The number of permutations of n distinct objects taken r at a time is ${}_nP_r = \frac{n!}{(n-r)!}$ - if not all of them are used   It is important to be aware that both n and r must be whole numbers.   r is the # that are used   Note that from this definition, the number of permutations of 7 distinct objects taken 7 at a time is ${}_7P_7 = \frac{7!}{(7-7)!} = \frac{7!}{0!}$. This must be equal to $7!$, so $0!$ must be defined to be equal to __1__.

6. From a group of 100 people, how many ways can a president, vice-president, and treasurer be selected?

$\underline{100}\ \underline{99}\ \underline{98}$ or ${}_{100}P_3$

7. Using Factorial Notation

Express as a single factorial: $12 \times 11 \times 10 \times 9!$ $12!$	Simplify i) $\frac{8!}{5!} = \frac{8 \times 7 \times 6 \times \cancel{5!}}{\cancel{5!}}$ $= 336$ ii) $\frac{n!}{n} = \frac{\cancel{n} \times (n-1)!}{\cancel{n}}$ $= (n-1)!$
Evaluate without your calculator ${}_8P_2$ $\underline{8}\ \underline{7}$ $= 56$	Show that $\underline{25}! + 24! = 26(24!)$ $25 \cdot 24! + 24!$ $(24!)(25+1)$ $26 \cdot 24!$

Express without using the factorial symbol

${}_{n}P_2$

$\underline{n} \cdot \underline{(n-1)}$

$n^2 - n$

Solve for n

${}_{n}P_2 = 42$

$\underline{n} \cdot \underline{(n-1)} = 42$

$n^2 - n = 42$

$n^2 - n - 42 = 0$

$(n-7)(n+6) = 0$

$n = 7$ or $\cancel{-6}$

but n must be a whole number.

Solve for n

${}_{n}P_3 = 720$

$\underline{n} \times \underline{n-1} \times \underline{n-2} = 720$

$n(n^2 - 3n + 2) = 720$

$n^3 - 3n^2 + 2n - 720 = 0$

by inspection (guess and check)

$n = 10$

Solve for n

${}_{10}P_n = 90$

$90 = \dfrac{10 \times 9 \cancel{\times 8 \times 7 \times 6!}}{\cancel{8!}}$

$90 = \underline{10} \times \underline{9}$

using 2 items

${}_{10}P_2$

$\boxed{n = 2}$

p524 # 1-8, 15, 22, 23, C1, C3