9.2 Analysing Rational Functions

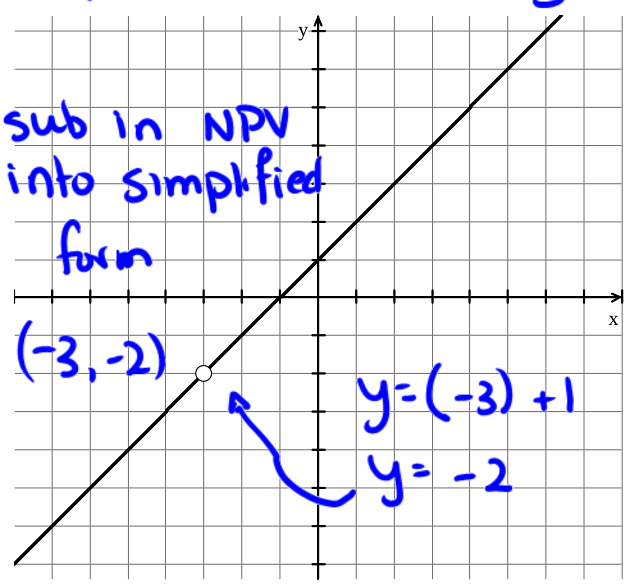
Example 1. Rational functions with points of discontinuity.

Examine the graphs of the functions below and analyse the behaviour near any non-permissible values. Identify the domain, range, asymptotes and any intercepts.

$$y=\frac{x^2+4x+3}{x+3}=\frac{\cancel{(x+3)}(x+1)}{\cancel{x+3}}$$

x+3's cancel out $\quad y=x+1$

→ point of discontinuity NPV $x\neq -3$



sub in NPV into simplified form

$(-3,-2)$

$y=(-3)+1$

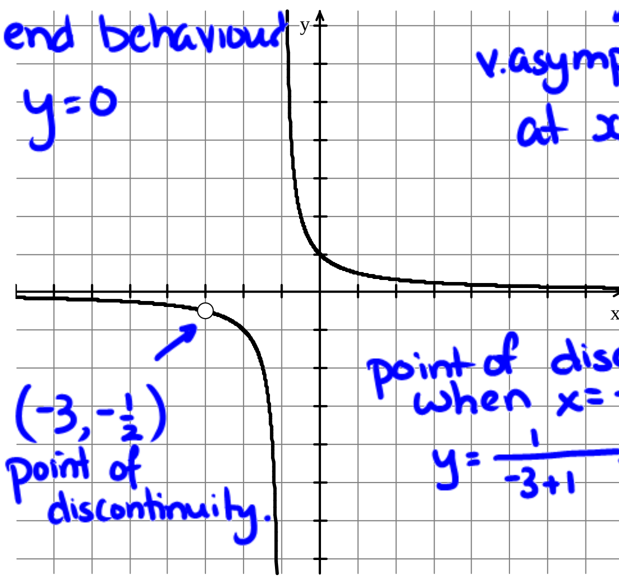
$y=-2$

$$y=\frac{x+3}{x^2+4x+3}$$

$$y=\frac{\cancel{x+3}}{\cancel{(x+3)}(x+1)}=\frac{1}{x+1}$$

v.asymptote at $x=1$

end behaviour $y=0$



$(-3,-\frac{1}{2})$ point of discontinuity.

point of disc when $x=-3$

$y=\frac{1}{-3+1}=\frac{1}{-2}$

At the non-permissible values, the graph of a rational function will have

- a vertical asymptote when denominator is zero and numerator is not zero factor in denominator does not cancel
- a point of discontinuity (hole) when denominator and numerator are zero simultaneously - NPV factor cancels out

To analyse the graph near a non-permissible value, use a table of values or the trace feature, or substitute into the simplified from.

x-int at -1 and 2

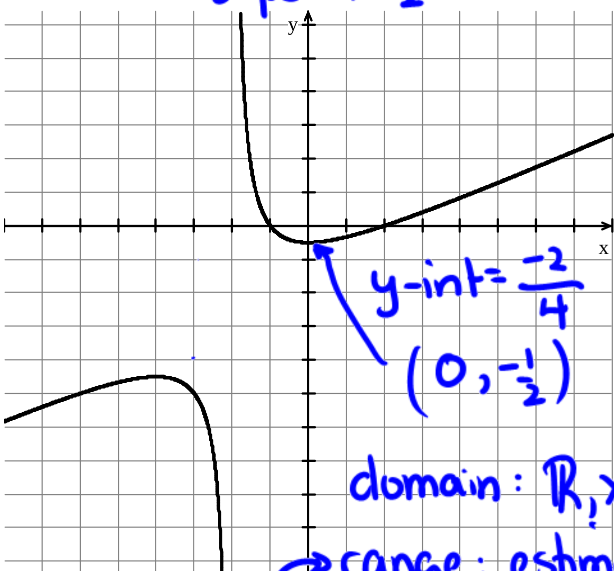
$$y=\frac{x^2-x-2}{2x+4}=\frac{(x-2)(x+1)}{2(x+2)}$$

end behaviour

$$y=\frac{x^2}{2x}=\frac{x}{2}\quad\boxed{=\frac{1}{2}x}$$

diagonal asymptote with slope $m=\frac{1}{2}$

v.asymptote $x=-2$



y-int $=\frac{-2}{4}$

$(0,-\frac{1}{2})$

domain: $\mathbb{R}, x\neq -2$

range: estimate or use technology

$$\boxed{y\geq -\frac{1}{2}\ \text{or}\ y\leq -4\tfrac{1}{2}}$$

$$y=\frac{4x}{x^2-9}$$

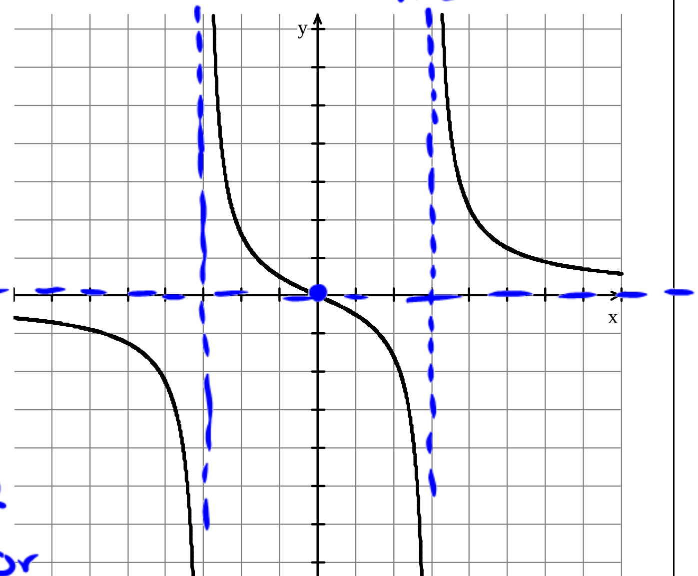
zero at $x=0$

NPV $x=3,-3$

end behaviour

$$y=\frac{4x}{x^2}=\frac{4}{x}\quad\boxed{y=0}$$

$x=-3$ $x=3$



Diagonal asymptotes exist when the degree of the numerator is 1 more than the degree of the ~~numerator~~ denominator. The slope of this asymptote can be found by dividing the coefficients of the highest degree terms in the numerator and denominator.

To find the zeros of a rational function, find the zeros of the numerator in the simplified form.

$$y=\frac{2x^3}{x^3-4x}=\frac{2x^3}{x(x^2-4)}=\frac{2x^3}{x(x+2)\underbrace{(x-2)}}$$

point of discontinuity

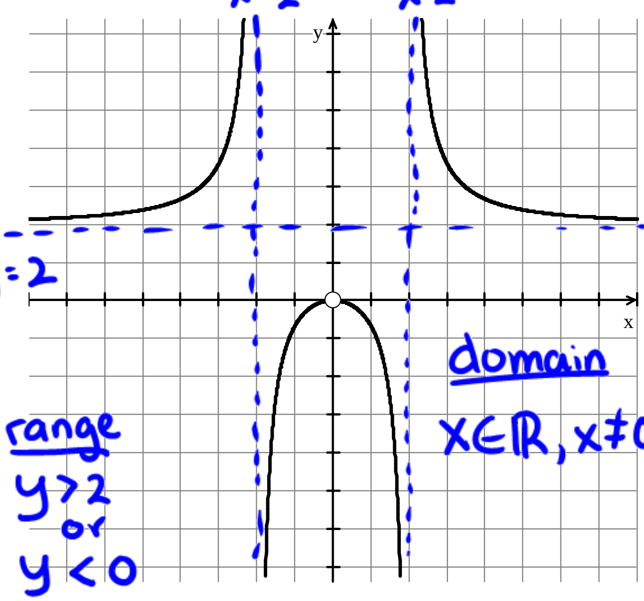
v.asympt.

end behaviour

$y=\frac{2x^3}{x^3}=\boxed{y=2}$

$x=-2$ $\quad$ $x=2$

$y=2$



domain

$x\in\mathbb{R}, x\neq 0, 2, -2$

range

$y>2$ or $y<0$

$$y=\frac{6x-6}{\underbrace{x^2+3}}$$

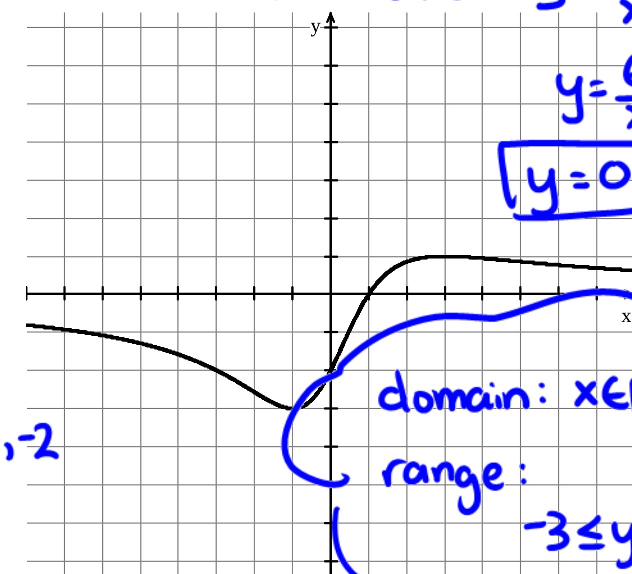
$6x-6=0 \rightarrow$ xint at $x=1$

no solution for x, so no NPV

end behaviour $y=\frac{6x}{x^2}$

$y=\frac{6}{x}$

$\boxed{y=0}$



domain: $x\in\mathbb{R}$

range: $-3\leq y\leq 1$

If there are no no-permissible values, the graph of a rational function will have no vertical asymptotes or points of discontinuity.

Horizontal asymptotes exist when the degree of the denominator $\geq$ degree of numerator

If the degree of the numerator is ~~equal to~~ less than the degree of the denominator, then the *x*-axis will be the horizontal asymptote.

if degree of numerator = degree of denominator then the h. asymptote is another value

if degree of num is 1 more than degree of denom = diagonal asymptote p451 #1-6, 8, 9, 12, 14-17, 21, 23

Test on Monday