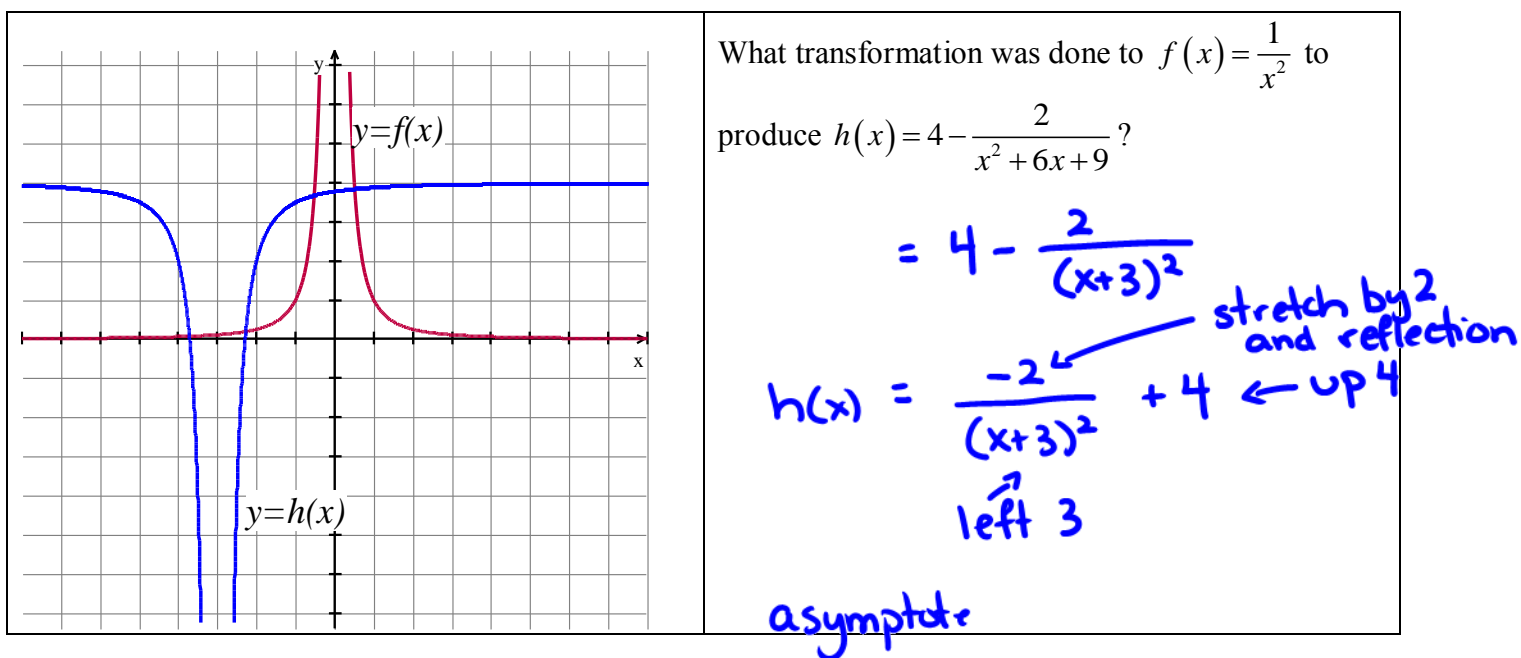
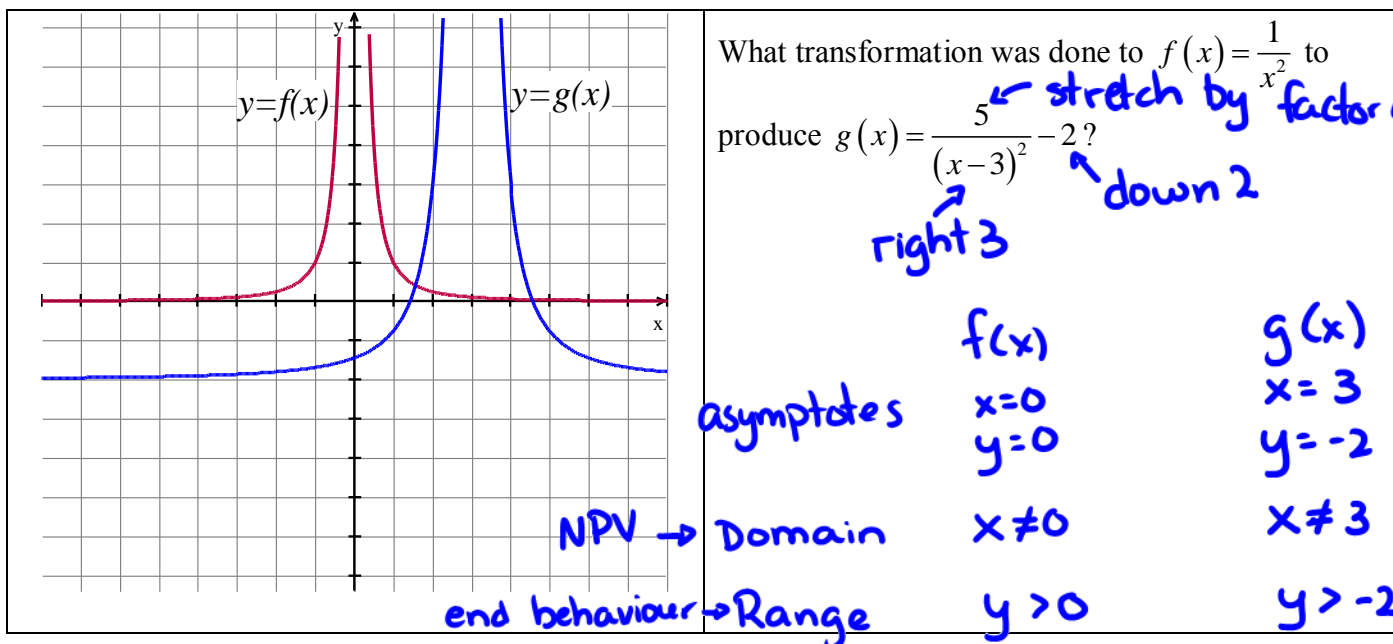


## 9.1B Rational Functions

Example 1. Given the functions  $f(x) = \frac{1}{x^2}$ ,  $g(x) = \frac{5}{(x-3)^2} - 2$  and  $h(x) = 4 - \frac{2}{x^2 + 6x + 9}$  and the graphs below. Explain how the graphs are related, and use your knowledge of transformations to explain this. Give the non-permissible values, behaviour near non-permissible values, domain and range, equations of any asymptotes and end behaviour.



Characteristic	$f(x) = \frac{1}{x^2}$	$g(x) = \frac{5}{(x-3)^2} - 2$	$h(x) = 4 - \frac{2}{x^2 + 6x + 9}$
Non-permissible value	$x \neq 0$	$x \neq 3$	$h(x) = \frac{-2}{(x+3)^2} + 4$ $x \neq -3$
Behaviour near non-permissible value	v. asymptote	v. asymptote	v. asymptote
End behavior	h. asymptote	h. asymptote	h. asymptote
Domain	$x \in \mathbb{R}$ $x \neq 0$	$x \in \mathbb{R}$ $x \neq 3$	$x \in \mathbb{R}$ $x \neq -3$
Range	$y > 0$	$y > -2$	$y < 4$
Vertical asymptote(s)	$x = 0$	translated 3 right $x = 3$	translated 3 left $x = -3$
Horizontal asymptote(s)	$y = 0$	translated 2 down $y = -2$	translated 4 up $y = 4$

Example 2. How does the graph of  $k(x) = 3 - \frac{1}{2x^2 - 20x + 50}$  compare to  $f(x) = \frac{1}{x^2}$ ?

$$= \frac{-1}{2(x^2 - 10x + 25)} + 3$$

v. reflection

$$= \frac{-1}{2(x-5)^2} + 3$$

h. stretch by  $\frac{1}{2}$       right 5      up 3

asymptotes  
 $x = 5$   
 $y = 3$

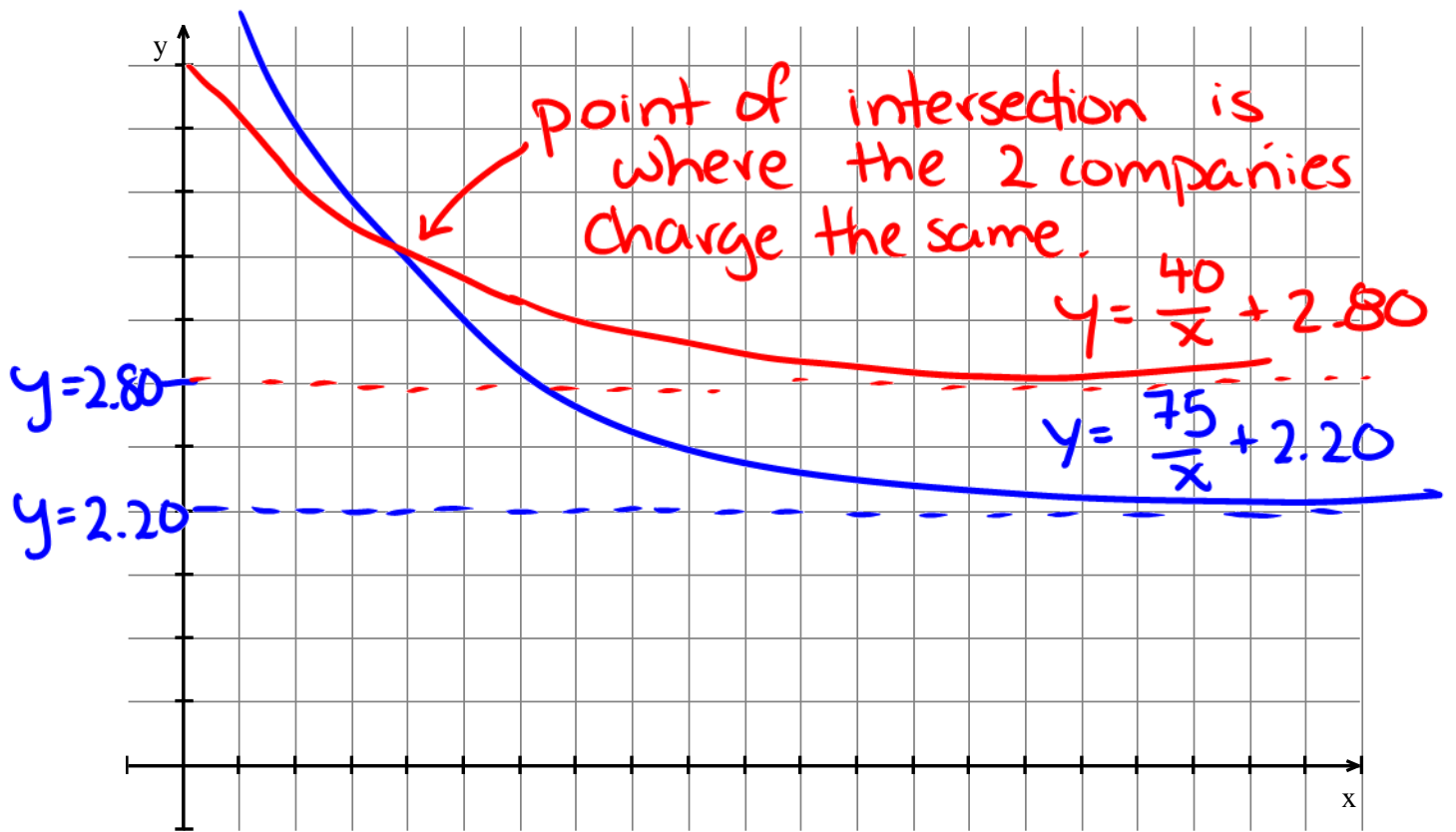
Example 3. The Math Department is going to print some booklets, and has price quotes from two companies

- Company A - \$40 setup fee and \$2.80 per booklet
- Company B - \$75 setup fee and \$2.20 per booklet

cost  
booklet

- Represent the average cost per booklet for each company as a function of the number of booklets printed, and then graph the two functions.
- Explain the characteristics of the graphs and their relation to the situation. What do the graphs show about how the average cost changes as the number of booklets printed increase?
- How should the Math Department choose a printing company?

Company A	Company B
$\text{Avg cost} = \frac{40 + 2.80x}{x}$ $= \frac{40}{x} + \frac{2.80x}{x}$ $C = \frac{40}{x} + 2.80$ <p> <math>\nearrow</math>                      v. asymptote  <math>x=0</math> </p> <p> <math>\uparrow</math>                      end behaviour  <math>y=2.80</math> </p>	$\text{Avg cost} = \frac{75 + 2.20x}{x}$ $C = \frac{75}{x} + 2.20$ <p> <math>\nearrow</math>                      v. asymptote  <math>x=0</math> </p> <p> <math>\uparrow</math>                      h. asympt.  <math>y=2.20</math> </p>



$$40 + 2.80x = 75 + 2.20x$$

$$.6x = 35$$

$$x = 58.3 \text{ books}$$

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