9.1A Rational Functions

The distance from Tsawwassen to Stanley Park is about 36 km. Copy and complete a table of values giving the time required to cycle this distance for a variety of speeds.

	Average Speed (km/h)	1	2	3	4	5	6	8	9	10	12	15	18	20	36
0															
np.	Time (h)	36	18	12	9	7.2	6		4		3		2		1

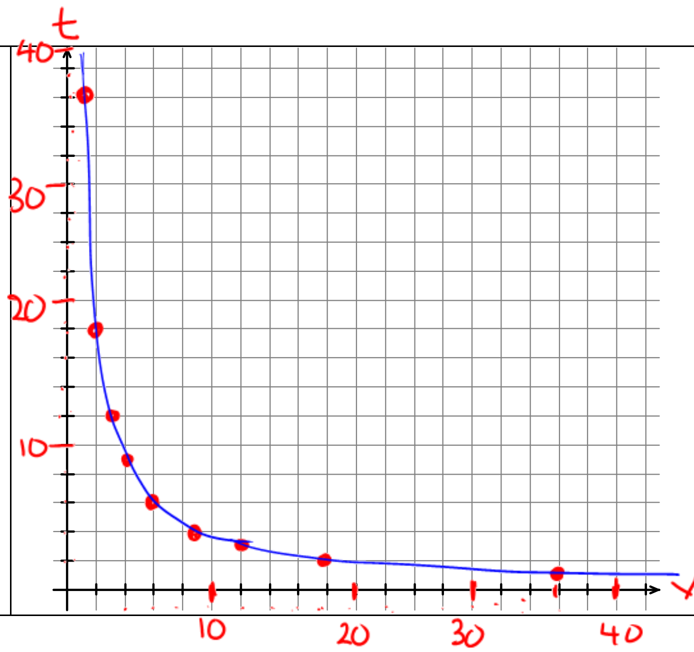
Write an equation to express the time t, in hours, as a function of the average speed v, in km/h.

$$t = \frac{36}{v}$$

Graph the function on the grid to the right.

What are the x and y intercepts? none

if $t=0$, you would need to be going infinitely fast



Make a table of values for the function $\boxed{y = \frac{1}{x}}$, and then use these values to sketch a graph of this function.

x	y
-4	$-\frac{1}{4}$
$-\frac{1}{2}$	-2
-1	-1
0	np
1	1
2	$\frac{1}{2}$
3	$\frac{1}{3}$

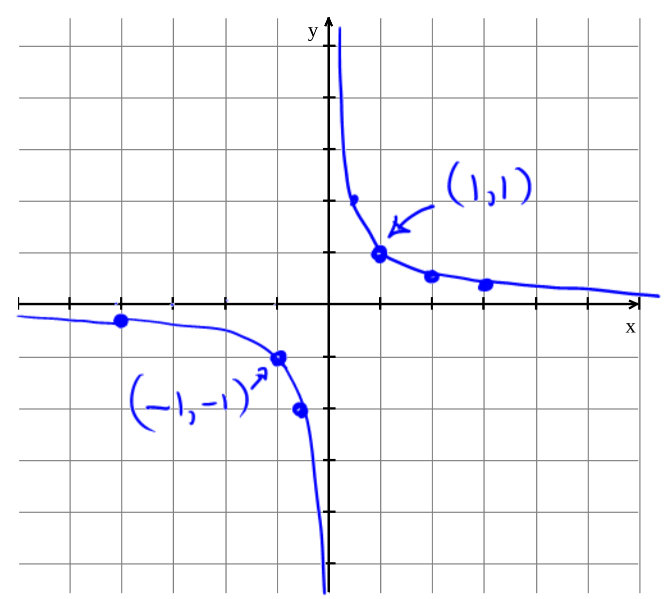
What is the behaviour of the function as x approaches zero?

v. asymptote

What are the x and y intercepts? Explain.

no x and y-intercepts

function is undefined for $x=0$ or $y=0$



Give the domain and range of the function, along with the equations of any asymptotes?

Domain: $x \neq 0$

Range: $y \neq 0$

asymptotes: $x=6$

$y=0$

Rational Functions

The numbers $-\frac{2}{3}$, 5, 0.8, $\frac{1}{8}$, $-0.1\overline{6}$, 0, 0.14783, -23456 are examples of rational numbers. A ***rational number*** is any number that can be expressed in the form $\frac{m}{n}$ where m and n are both integers, and $n \neq 0$.

A ***rational function*** is any function of the form $f(x) = \frac{p(x)}{q(x)}$ where $p(x)$ and $q(x)$ are both polynomial functions and $q(x) \neq 0$.

Examples of rational functions:

rational function that has a reciprocal polynomial

that is a polynomial

$$y = \frac{x^2+1}{x^3-7x-2} \qquad f(x) = \frac{1}{x^2-4} \qquad f(x) = \frac{2x+1}{3x-5} \qquad g(x) = \frac{x^4+10x-17}{x^7-6x^5+x^2-5x+2} \qquad y = x^2+7x-2$$

Note: All polynomial functions are also rational functions
The reciprocal of a polynomial function is also a rational function.

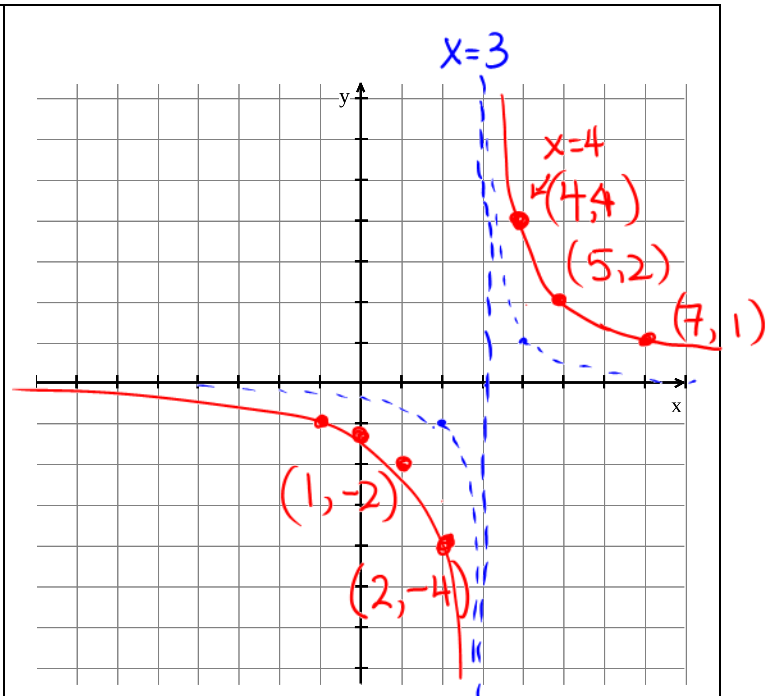
Example 1. What transformations are applied to the graph of $y = \frac{1}{x}$ to produce the graphs of the following functions. Give the domain and range of the transformed function, the equations of any asymptotes, and any non-permissible values. Determine the x and y intercepts if any. What happens to the graph as $|x|$ becomes very large? Sketch a graph of the transformed function.

$y = \frac{1}{x} \longrightarrow y = \frac{4}{x-3}$

$x \to x-3$ move 3 right

$y = \frac{1}{x} \to y = 4 \cdot \frac{1}{x}$ v. stretched by 4

* substituting values for x can help you plot points.



asymptotes

$x=3$

$y=0$

domain $x \neq 3$

range $y \neq 0$

$\boxed{\text{y-intercept}}$

$y = \frac{4}{(0)-3} = \frac{-4}{3}$

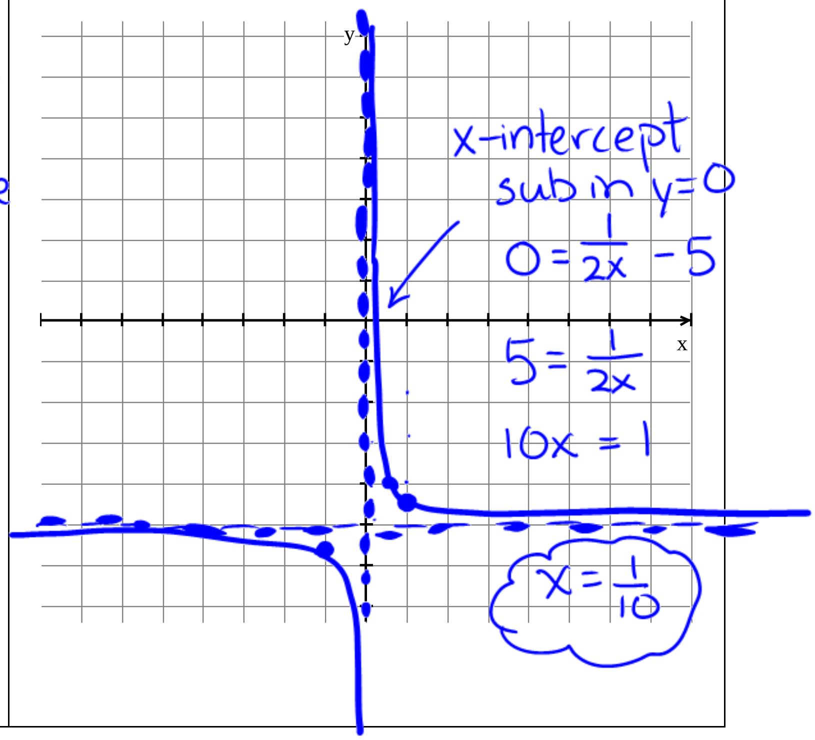
$$y = \frac{1}{2x} - 5$$

h. stretch by $\frac{1}{2}$
asymptote does not move

move down 5, so h. asymptote changes

asymptotes : $y = -5$
$x = 0$

domain : $x \neq 0$
range $y \neq -5$



x-intercept
sub in $y = 0$

$0 = \frac{1}{2x} - 5$

$5 = \frac{1}{2x}$

$10x = 1$

$x = \frac{1}{10}$

$$y = \frac{-2}{x+6} - 5$$

Vertical stretch by 2 with reflection

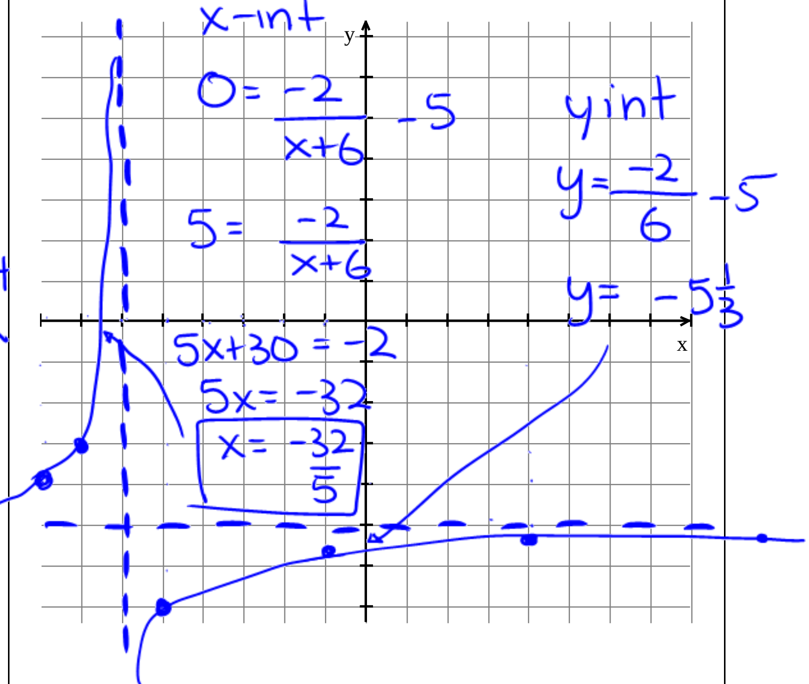
left 6

down 5

x	y
-5	-7
-1	-5.4
4	-5.2
-8	-4
-7	-3

asymptotes
$x = -6$
$y = -5$

domain $x \neq -6$
range $y \neq -5$



x-int

$0 = \frac{-2}{x+6} - 5$

$5 = \frac{-2}{x+6}$

$5x + 30 = -2$

$5x = -32$

$\boxed{x = \frac{-32}{5}}$

y int

$y = \frac{-2}{6} - 5$

$y = -5\frac{1}{3}$

Example 2. Graphing Rational functions of the form $y = \frac{linear\ \ function}{linear\ \ function}$

v. stretch by 7 and reflected.

up 2

Graph the function $y = \frac{2x-3}{x+2}$. $= 2 + \frac{-7}{x+2}$ $= \frac{-7}{x+2} + 2$

synthetic division.

$$\begin{array}{r|rr} -2 & 2 & -3 \\ & \downarrow & -4 \\ \hline & 2 & -7 \end{array}$$

← remainder

left 2

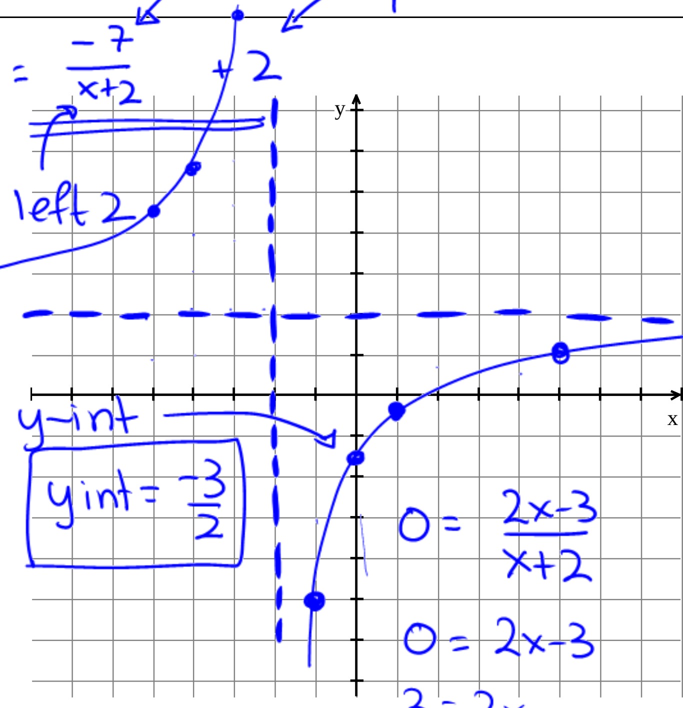
asymptotes

$x = -2$

$y = 2$

domain $x \neq -2$

range $y \neq 2$



x	y
-1	-5
0	-1.5
1	$-.\overline{3}$
-5	1
-3	9
-4	5.5
-5	$4.\overline{3}$

$0 = \frac{2x-3}{x+2}$

$0 = 2x - 3$

$3 = 2x$

$\boxed{x = \frac{3}{2}}$

Example 3. P442 #1-8

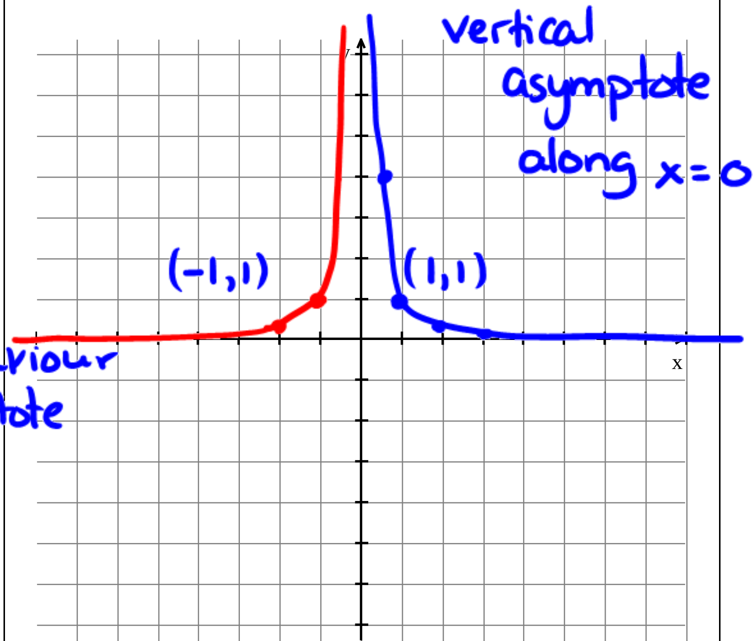
Graph the function $y = \frac{1}{x^2}$.

x	y
1	1
2	$\frac{1}{4}$
3	$\frac{1}{9}$
∞	almost 0
.5	4
-1	1
-2	$\frac{1}{4}$

end behaviour

h. asymptote

$y = 0$



vertical asymptote along $x = 0$